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Mechatronics Engineering

1a) A differential equation is a relationship between an independent variable 'x' and dependent variable 'y' and one or more derivative of y with respect to x
E.g (i) $\frac{dy}{dx} = y + \frac{y}{x}$

(B) $y = Ae^{-4x} + Be^{-6x}$

A second order differential equation

This is because it contains two variables

$$y = Ae^{-4x} + Be^{-6x}$$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (2)}$$

Solve eqn (1) and (2) simultaneously

Multiply eqn (1) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{--- (3)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (4)}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$A = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} \quad \text{--- (5)}$$

Subs. eqn (5) into eqn (1)

$$\frac{dy}{dx} = 4 \left(\frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2

$$2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B$$

$$B = \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

Sub A and B into the degenerate equation.

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{18 \frac{dy}{dx} + 3 \frac{d^2y}{dx^2} - 8 \frac{dy}{dx} - 2 \frac{d^2y}{dx^2}}{-24}$$

$$y = \frac{10 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-24}$$

$$-24y = 10 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$