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1.2) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x

$$\text{Examples: (i) } \frac{dy}{dx} = 2 + \frac{y}{x}$$

$$(ii) \frac{dy}{dx} = y + \frac{y}{x}$$

$$y = Ae^{-4x} + Be^{-6x}$$

2) A second order differential equation

A second order differential equation can be formed because it contains 2 constants in the degenerate equation

$$y = Ae^{-4x} + Be^{-6x}$$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \dots \textcircled{1}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \dots \textcircled{2}$$

Solving equ ① and ② Simultaneously

Multiply equ ① by 6

$$6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \dots \textcircled{3}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \dots \textcircled{4}$$

$$\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$A = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \dots \textcircled{5}$$

Substituting equ ⑤ into equ ②

$$\frac{dy}{dx} = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 6Be^{-6x}}{2}$$

Multiply through by 2

$$2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B \quad \therefore \quad \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} =$$

Substitute A and B into the degenerate equation

$$\therefore y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2} + e^{-4x}}{-8e^{-4x}} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2} + e^{-6x}}{12e^{-6x}}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{-72 \frac{dy}{dx} - 12 \frac{d^2y}{dx^2} + 32 \frac{dy}{dx} + 8 \frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}}{96}$$

96