2) if $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t} k$. Determine (a) dr/dt (b) $\frac{d^2r}{dt^2}$ (c) $\left|\frac{d^2r}{dt^2}\right|$

Solution

a) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t j + 6e^{2t} k$

b) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} K$

c) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$ $\therefore 2i\ [18\sin(3)(0)]j + 12e^{2(0)}k$

$= 2i + 18\sin 0 j + 12e^0 k$

$= 2i + 18(0)j + 12(1)k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{4+144} = \sqrt{148} = 12.17$

① $r(x)$ ...

Solution

① $\frac{d\vec{r}}{dx} = 2xi + (3)j + \cos x\, k$

$\therefore \frac{dr}{dx} = 2xi + 3j + \cos x\, k$

③ $\frac{d^2r}{dx^2} = 2i - \sin x\, k$

③ $\left|\frac{dr}{dx}\right| = \sqrt{(2x)^2 + 3^2 + (\cos x)^2}$

$= \sqrt{4x^2 + 9 + \cos^2 x}$

At $x = 1 \rightarrow \sqrt{4(1)^2 + 9 + \cos^2(1)}$

$= \sqrt{4+9+0.999} = \sqrt{13.999} = 3.74$

④) $\frac{d}{dx}(r \cdot r) \Rightarrow (r \cdot r) = [(x^2 i) + (3x+2)j + \sin x k] \cdot [(x^2)i + (3x+2)j$
$+ \sin x k]$

$= x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$

$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$

$\frac{d}{dx}(r \cdot r) = 4x^3 + 18x + 12 + 2\sin x \cos x$

$\frac{d}{dx}(r \cdot r)$ at $x=1$ $= 4(1)^3 + 18(1) + 12 + 2\sin(1)\cos(1)$

$= 4 + 18 + 12 + 2(0.0175)(0.999)$