Assignment 1

Francis Chinaeme Ikenna

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Define a differential equation and give two examples

a A differential equation is a relationship between an independent variable, a dependent variables, y and one or more derivatives of y with respect to x

Example: i) $xy\frac{d^2y}{dx^2} + y\frac{dy}{dx} + e^{2x} = 0$

ii) $y\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 14x^2 = \sin 4y$

An expression has been obtained for an engineering system to be given in equation

$y = Ae^{-4x} + Be^{-6x} \quad -(i)$

b i What is the order of the differential equation that can be formed from the expression: It is second order equation

b ii Give a reason for your answer in b(i) It is because

b iii From a differential equation from the expression

$y = Ae^{-4x} + Be^{-6x} \quad -(i)$

$dy/dx = -4Ae^{-4x} - 6Be^{-6x} \quad -(ii)$

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x} \quad -(iii)$

$4Ae^{-4x} = -dy/dx - 6Be^{-6x}$

$A = \left(-\frac{dy}{dx} - 6Be^{-6x}\right) \frac{1}{4e^{-4x}} \quad -(iv)$

Sub (iv) & (iii)

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$

$d^2y/dx^2 = 16\left(-\frac{dy}{dx} - 6Be^{-6x}\right) \cdot \frac{1}{4e^{-4x}} \cdot e^{-4x} + \left(\frac{1}{4}\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)\frac{1}{36e^{-6x}} + 36Be^{-6x}$

$y = \frac{1}{4}\left(-\frac{dy}{dx} - \frac{d^2y}{2dx^2}\right) + \frac{1}{12}\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)$

4-

$d^3y/dx^3 = 16(-dy/dx - 6e^{-6x}) + 4e^{-4x} \cdot e^{-4x} + 256e^{-4x}$

$d^3y/dx^3 = 16(-dy/dx - 6e^{-6x}/4) + 256e^{-4x}$

$d^3y/dx^3 = -4\,dy/dx - 24e^{-6x} + 256e^{-4x}$

$d^3y/dx^3 = -4\,dy/dx + 12e^{-6x}$

$6 = \left(\frac{d^3y}{dx^3} + 4\frac{dy}{dx}\right) \cdot \frac{1}{12e^{-4x}}$ ---(v)

$A = \left(-\frac{dy}{dx} - 6\left(\frac{d^2y}{2dx^2} + 4\frac{dy}{2dx}\right) \cdot \frac{1}{12e^{-4x}}\right) e^{-6x} \cdot \frac{1}{4x}$

$B = \left(-\frac{dy}{dx} - \frac{6}{2}\frac{d^2y}{dx^2} - 2\frac{dy}{2dx}\right) \frac{1}{4e^{-4x}}$ ---(vi)

put equ (v) & (vi) and equ (iv)

$y = Ae^{-4x} + Bx^{-6x}$ ---(a)

$y = \left(-\frac{3dy}{dx} - \frac{d^2y}{2dx^2}\right) \cdot \frac{1}{4}e^{-4x} \cdot e^{-4x} + \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)\frac{1}{12}e^{-4x}e^{-4x}$

$y = \frac{1}{4}\left(-\frac{3dy}{dx} - \frac{d^2y}{2dx^2}\right) + \frac{1}{12}\left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx}\right)$

$y = \frac{-3dy}{4dx} - \frac{d^2y}{8dx^2} + \frac{d^2y}{12dx^2} + \frac{4dy}{12dx}$

$y = \frac{-5dy}{12dx} - \frac{d^2y}{24dx}$

$y = \left(-\frac{5dy}{dx} - \frac{d^2y}{24dx^2}\right) + \frac{1}{12}$