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ENG 282 [Engineering mathematics II].

1a) Define differential Equation and give two examples

Answers.

A differential Equation is a relationship between an independent variable, x, a dependent variable y, and one or more derivatives of y with respect to x.

E.g. ~~$x^2 dy/dx$=~~

(i) $x\,dy/dx - y^2 = 0$.

(ii) $xy\,d^2y/dx^2 + y\,dy/dx + e^{3x} = 0$.

(b) An expression has been obtained for an engineering system to be as given in Equation (i).

$y = Ae^{-4x} + Be^{-6x}$.

(i) What is the order of the differential equation that can be formed from the expression.

- 2nd Order Differential Equation.

(ii) Give a reason for your answer in b(i).

- This is due to the function ~~of~~ with two arbitrary constants A and B.

(iii) To form a differential Equation.

$y = Ae^{-4x} + Be^{-6x}$ ... (i)

Remember to differentiate twice.

$dy/dx = -4Ae^{-4x} - 6Be^{-6x}$ ... (ii)

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$ ... (iii)

To find the constant A.

Multiply equation (ii) by 6

$6\,dy/dx = -24Ae^{-4x} - 36Be^{-6x}$.

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$ ... (iv)

Using the elimination method, add equation (iii) from equation (iv)

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$6\,dy/dx + d^2y/dx^2 = -24Ae^{-4x} + 16Ae^{-4x}$.

$= 6\,dy/dx + d^2y/dx^2 = -24Ae^{-4x} + 16Ae^{-4x}$.

$= 6\,dy/dx + d^2y/dx^2 = -8Ae^{-4x}$

Now, making A subject of formula.

Divide through by $-8Ae^{-4x}$

$$\frac{-1}{8e^{-4x}}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] = A.$$

To find the constant B.

Multiply equation (ii) and (iv), ~~4~~

$4\,dy/dx = -16Ae^{-4x} - 6Be^{-6x}$ ... (v)

Bringing Equation (iii) and (v) together.

$4\,dy/dx = -16Ae^{-4x} - 24Be^{-6x}$

$d^2y/dx^2 = 16Ae^{-4} + 36Be^{-6x}$

Adding both Equations.

$4\,dy/dx + d^2y/dx = -24Be^{-6x} + 36Be^{-6x}$

$4\,dy/dx + d^2y/dx^2 = 12Be^{-6x}$

Making B, subject of formula.

$$\frac{1}{12e^{-6x}}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] = B$$

Putting A and B into equation (1).

$$y = \frac{-1}{8e^{-4x}}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]e^{-4x} + \frac{1}{12e^{-6x}}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]e^{-6x}$$

$$y = \frac{-1}{8}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] + \frac{1}{12}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right].$$

Expanding the brackets.

$$y = \frac{-6}{8}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{4}{12}\frac{dy}{dx} + \frac{1}{12}\,d^2y/dx^2$$

Collect like terms!

$$y = -\frac{3}{4}\,dy/dx - \frac{1}{8}\,d^2y/dx^2 + \frac{1}{3}\,dy/dx + \frac{1}{12}\,d^2y/dx^2$$

Collect like terms.

$$y = -\frac{3}{4}\,{}^{dy}/_{dx} + {}^{1}/_{3}\,{}^{dy}/_{dx} - {}^{1}/_{8}\,{}^{d^2y}/_{dx^2} + {}^{1}/_{12}\,{}^{d^2y}/_{dx^2}$$

$$= -\frac{5}{12}\,{}^{dy}/_{dx} - {}^{1}/_{24}\,{}^{d^2y}/_{dx^2}$$

Multiply through by 24.

$$24y = -10\,{}^{dy}/_{dx} - {}^{d^2y}/_{dx^2}.$$