

1b) Define differential equation and give two examples

Answers:

A differential equation is a relationship between an independent variable, x , a dependent variable y , and one or more derivatives of y with respect to x .

eg. $x \frac{dy}{dx} =$

(i) $x \frac{dy}{dx} - y^2 = 0$

(ii) $2y \frac{d^2y}{dx^2} + y \frac{dy}{dx} + e^{3x} = 0$

(b) An expression has been obtained for an engineering system to be as given in equation (i).

$$y = Ae^{-4x} + Be^{-6x}$$

(i) What is the order of the differential equation that can be formed from the expression.

- 2nd Order Differential Equation

(ii) Give a reason for your answer in (i).

- This is due to the function with two arbitrary constants A and B .

(iii) To form a differential equation

$$y = Ae^{-4x} + Be^{-6x} \quad \dots (i)$$

Remember to differentiate twice

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \dots (ii)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \dots (iii)$$

To find the constant A .

Multiply equation (ii) by 6

$$6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \dots (iv)$$

Using the elimination method, add equation (iii) from equation (iv)

$$6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -24Ae^{-4x} + 16Ae^{-4x}$$

$$= 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -24Ae^{-4x} + 16Ae^{-4x}$$

$$= 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

Now, making A subject of formula.

Divide through by $-8Ae^{-4x}$

$$\frac{-1}{8e^{-4x}} \left[6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] = A$$

To find the constant B.

Multiply equation (ii) and (iv),

$$4 \frac{dy}{dx} = -16Ae^{-4x} - 6Be^{-6x} \dots (v)$$

Bringing Equation (iii) and (v) together.

$$4 \frac{dy}{dx} = -16Ae^{-4x} - 6Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

Adding both equations.

$$4 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -24Be^{-6x} + 36Be^{-6x}$$

$$4 \frac{dy}{dx} + \frac{d^2y}{dx^2} = 12Be^{-6x}$$

Making B, subject of formula.

$$\frac{1}{12e^{-6x}} \left[4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] = B$$

Putting A and B into equation (i).

$$y = \frac{-1}{8e^{-4x}} \left[6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] e^{-4x} + \frac{1}{12e^{-6x}} \left[4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] e^{-6x}$$

$$y = \frac{-1}{8} \left[6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] e^{-4x} + \frac{1}{12} \left[4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \right] e^{-6x}$$

Expanding the brackets.

$$y = \frac{-6}{8} \frac{dy}{dx} - \frac{1}{8} \frac{d^2y}{dx^2} + \frac{4}{12} \frac{dy}{dx} + \frac{1}{12} \frac{d^2y}{dx^2}$$

Collect like terms!

$$y = \frac{-3}{4} \frac{dy}{dx} - \frac{1}{8} \frac{d^2y}{dx^2} + \frac{1}{3} \frac{dy}{dx} + \frac{1}{12} \frac{d^2y}{dx^2}$$

Collect like terms.

$$y = -\frac{3}{4} \frac{dy}{dx} + \frac{1}{3} \frac{dy}{dx} - \frac{1}{8} \frac{d^2y}{dx^2} + \frac{1}{12} \frac{d^2y}{dx^2}$$

$$= -\frac{5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2}$$

Multiply through by 24.

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$