

1) Given that

$$f = x^2i + (3x+2)j + \sin xk$$

find

$$a) \frac{df}{dx} = 2xi + 3j + \cos xk$$

At $x=1$

$$\frac{df}{dx} = 2\cos i + 3j + \cos(1)k$$

$$b) \frac{d^2f}{dx^2} = \frac{d}{dx} \left[\frac{df}{dx} \right] = 2i - \sin xk$$

at $x=1$

$$\frac{d^2f}{dx^2} = 2i - \sin(1)k$$

$$= 2i - 0.0175k$$

$$c. \left[\frac{d^2f}{dx^2} \right] = 2i + 3j + 0.9998k$$

$$= [2^2 + 3^2 + 0.9998^2]^{1/2}$$

$$= 3.74$$

d. $\frac{d}{dx} (f \cdot f)$

$$f \cdot f = (x^2i + (3x+2)j + \sin xk) \cdot (x^2i + (3x+2)j + \sin xk)$$

$$= x^4 + (9x^2 + 12x + 4) - (\sin^2 x)$$

Remember

$$i \cdot i = j \cdot j = k \cdot k = 1$$

$$\frac{d}{dx} (f \cdot f) = 4x^3 + (18x + 12) + 2\sin x \cos x \text{ at } x=1$$

$$\frac{d}{dx} (f \cdot f) = 4 + 18 + 12 + 2\sin(1)\cos(1)$$

$$= 4 + 30 + 0.035$$

$$= 34.035$$

2) If

$$r = (t^2 + 3t)i - 2\sin t j + 3e^{2t} k$$

determine

$$a) \frac{dr}{dt} = (2t + 3)i - 2\cos t j + 6e^{2t} k$$

at $t=0$

$$\begin{aligned} \frac{ds}{dt} &= (2\cos 3t) \mathbf{i} - 6\cos 3t \mathbf{j} + 6e^{2t} \mathbf{k} \\ &= 3\mathbf{i} + 3\mathbf{i} - 6\mathbf{j} + 6\mathbf{k} \end{aligned}$$

$$\textcircled{b} \quad \frac{d}{dt} \left(\frac{ds}{dt} \right) = 2\mathbf{i} + 18\sin 3t \mathbf{j} + 12e^{3t} \mathbf{k}$$

at $t=0$

$$\begin{aligned} \frac{d^2s}{dt^2} &= 2\mathbf{i} + 18\sin 3(0) \mathbf{j} + 12e^{3(0)} \mathbf{k} \\ &= 2\mathbf{i} + 12\mathbf{k} \end{aligned}$$

$$\begin{aligned} \textcircled{c} \quad \left| \frac{d^2s}{dt^2} \right| &= \sqrt{2^2 + 12^2} \\ &= \sqrt{4 + 144} \\ &= \sqrt{148} \\ &= \sqrt{4 \times 37} \\ &= \sqrt{4} \times \sqrt{37} \\ &= 2\sqrt{37} \end{aligned}$$