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1a. A differential equation is a relationship between an independent variable(x) & a dependent variable (y) & one or more derivatives of y with respect to x.

Examples: i) $dy/dx = y + y/x$

ii) $dy/dx = 2 + y/x$

b. $y = Ae^{-4x} + Be^{-6x}$

i. A Second order differential equation.

ii. A Second order differential equation can be formed because there are two constants in the degenerate equation.

iii. $y = Ae^{-4x} + Be^{-6x}$

Soln,

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{---- (i)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (ii)}$$

Solving eqn(i) & (ii) Simultaneously
multiply eqn (i) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{---- (iii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \quad \text{---- (iv)}$$

Substituting eqn(iv) into eqn(i)

$$\frac{dy}{dx} = 4\left(\frac{6\,dy/dx + d^2y/dx^2}{8}\right) e^{-4x} - 6Be^{-6x}$$

B) $\frac{dy}{dx} = \frac{6\,dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$

Multiply through by 2.

$$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\,dy/dx - d^2y/dx^2}{-12e^{-6x}} \div B = 0$$

$$\frac{4\,dy/dx + d^2y/dx^2}{12e^{-6x}} = B$$

Substitute A & B into the degenerate equation

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \times e^{-4x} + \frac{4\,dy/dx + d^2y/dx^2}{12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8} + \frac{4\,dy/dx + d^2y/dx^2}{12}$$

$$y = \frac{-72\,dy/dx - 12\,d^2y/dx^2 + 32\,dy/dx + 8\,d^2y/dx^2}{96}$$

$$y = \frac{-40\,dy/dx - 4\,d^2y/dx^2}{96}$$

$$96y = -40\,dy/dx - 4\,d^2y/dx^2$$

$$24y = -10\,dy/dx - d^2y/dx^2$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$