OGUNUEGBUNAM FAVOUR CHIDI
17/ENG01/021
CHEMICAL ENGINEERING
ENG 282

(a) Define differential Equation and gives two example.

Answer

A differential equation is a relationship between an independent variable x, a dependent variable y, and one or more derivatives of y with respect to x.

Eg

(i) $(x+1)^2 dx + y = 0$

(ii) $x\, dy/dx = 5x^3 + 4$

(b) An expression has been obtained for an engineering system to be as given in Equation (i)

$$y = Ae^{4x} + Be^{-6x}$$

(i) What is the order of the differential equation that can be formed from the expression.

- 2nd Order differential Equation

(ii) Give a reason for your answer in (i)

This is due to the function with two arbitrary constant A and B.

(iii) To form a differential Equation

$$y = Ae^{-4x} + Be^{-6x}$$

$$dy/dx = -4Ae^{-4x} - 6Be^{-6x} \quad \text{— (ii)}$$

$$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x} \quad \text{— (iii)}$$

To find the constant A

multiply equation (ii) by 6

$$6\, dy/dx = -24Ae^{-4x} - 36Be^{-6x} \quad \text{— (iv)}$$

$$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$$

Using elimination method, add equation (iii) to (iv)

$$6\, dy/dx + d^2y/dx^2 = -24Ae^{-4x} + 16Ae^{-4x} - 36Be^{-6x} + 36Be^{-6x}$$

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -24Ae^{-4x} + 16Ae^{-4x}$

$\Rightarrow 6\,dy/dx + d^2y/dx^2 = -8Ae^{-4x}$

Making A subject of Formula

divide through by $-8e^{-4x}$

$$\frac{-1}{8e^{-4x}}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] = A$$

To find the constant B

multiply equation (i) by 4

$$4\,dy/dx = -16Ae^{-4x} - 24Be^{-6x} \quad \text{---- (v)}$$

Bringing equation (iii) and (iv) together

$$4\frac{dy}{dx} = -16Ae^{-4x} - 24Be^{-6x}$$

$$d^2y/dx^2 = 16Ae^{-4} + 36Be^{-6x}$$

Adding both equations

$$4\,dy/dx + d^2y/dx^2 = -24Be^{-6x} + 36Be^{-6x}$$

$$4\,dy/dx + d^2y/dx^2 = 12Be^{-6x}$$

Making B subject of formula

$$\frac{1}{12e^{-6x}}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] = B$$

Putting A and B into equation (i)

$$y = -\frac{1}{8e^{-4x}}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]e^{-4x} + \frac{1}{12e^{-6x}}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]e^{-6x}$$

$$y = -\frac{1}{8}\left[6\frac{dy}{dx} + \frac{d^2y}{dx^2}\right] + \frac{1}{12}\left[4\frac{dy}{dx} + \frac{d^2y}{dx^2}\right]$$

Expanding the bracket

$$y = -\frac{6}{8}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{4}{12}\frac{dy}{dx} + \frac{1}{12}\frac{d^2y}{dx^2}$$

$$y = -\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{3}\frac{dy}{dx} + \frac{1}{12}\frac{d^2y}{dx^2}$$

Collecting like terms

$$y = -\frac{3}{4}\frac{dy}{dx} + \frac{1}{3}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{12}\frac{d^2y}{dx^2}$$

$$y = \frac{-9+4}{12}\frac{dy}{dx} - \frac{3+2}{24}\frac{d^2y}{dx^2}$$

$$y = \frac{-5}{12}\frac{dy}{dx} - \frac{1}{24}\frac{d^2y}{dx^2}$$

Multiply through by 24

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}.$$