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Assignment I

a) Define differential equation and give two examples

- A differential equation is a relationship between an independent variable, x , a dependent variable, y , and one or more derivatives of y with respect to x .

Example

$$(i) xy \frac{d^2y}{dx^2} + y \frac{dy}{dx} + e^{3x} = 0$$

$$(ii) \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y = \sin 2x$$

b) An expression has been obtained for an engineering system in equation (i)

$$y = Ae^{-4x} + Be^{-6x} \quad \text{--- (1)}$$

i) What is the order of the differential equation that can be formed from the expression?

- Second order equation

(ii) Given, state a reason for your answer in (i)

-- It is because a function with two arbitrary constants gives a 2nd order equation.

(iii) Form a differential equation from the expression.

$$y = Ae^{-4x} + Be^{-6x} \quad \text{--- (1)}$$

$$\frac{dy}{dx} = -4Ae^{-4x} + (-6)Be^{-6x} \quad \text{--- (ii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (iii)}$$

From eqn (iii)

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$$

$$4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$A = \left(-\frac{dy}{dx} - 6Be^{-6x} \right) \cdot \frac{1}{4e^{-4x}} \quad \text{--- (iv)}$$

Substitute eqn (iv) in eqn (iii)

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16 \left(-\frac{dy}{dx} - 6Be^{-6x} \right) \frac{e^{-4x}}{4e^{-4x}} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$B = \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \cdot \frac{1}{12e^{-6x}} \quad \text{--- (v)}$$

Substitute eqn (v) in eqn (iv)

$$A = \left(-\frac{dy}{dx} - 6 \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \frac{e^{-6x}}{12e^{-6x}} \right) \cdot \frac{1}{4e^{-4x}}$$
$$= \left(-\frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} - 2\frac{dy}{dx} \right) \cdot \frac{1}{4e^{-4x}} \quad \text{--- (vi)}$$

Put eqn (vi) and eqn (v) in eqn (i)

$$y = \left(-\frac{3}{4} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2} \right) \frac{e^{-4x}}{4e^{-4x}} + \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \frac{e^{-6x}}{12e^{-6x}}$$

$$y = -\frac{3}{4} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2} + \frac{1}{12} \frac{d^2y}{dx^2} + \frac{1}{3} \frac{dy}{dx}$$

$$= \frac{-5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2}$$

$$y = \left(-\frac{5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2} \right) \cdot \frac{1}{12}$$