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CIVIL ENGINEERING

17/ENG03/038

(a) A differential equation is a relationship between an independent variable x and a dependent variable y and one or more derivative of y with respect to x

$$\text{Eg } \frac{dy}{dx} = y + yx^{-1}$$

$$b) y = Ae^{-4x} + Be^{-6x}$$

It is a second order differential equation because it contains two arbitrary constants.

Solution

$$\frac{dy}{dx} = -4x Ae^{-4x} - 6x Be^{-6x} \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} = +16x^2 Ae^{-4x} + 36x^2 Be^{-6x} \quad \text{--- (2)}$$

Solving equation (1) and (2) simultaneously

Multiply eqn (1) by $6x$ and eqn (2) by 1

$$6x \frac{dy}{dx} = -24x^2 Ae^{-4x} - 36x^2 Be^{-6x} \quad \text{--- (3)}$$

$$\frac{d^2y}{dx^2} = 16x^2 Ae^{-4x} + 36x^2 Be^{-6x} \quad \text{--- (4)}$$

$$6x \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8x^2 Ae^{-4x}$$

$$A = \frac{6x \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8x^2 e^{-4x}} \quad \text{--- (5)}$$

Substituting eqn (5) into eqn (1)

$$\frac{dy}{dx} = \frac{-4x}{-8x^2 e^{-4x}} \left(6x \frac{dy}{dx} + \frac{d^2y}{dx^2} \right) e^{-4x} - 6x Be^{-6x}$$

$$\frac{dy}{dx} = \left(\frac{6xy/dx + d^2y/dx^2}{2x} \right) - 6xBe^{-6x}$$

Multiply through by 2

$$2 \frac{dy}{dx} = 6xy/dx + d^2y/dx^2 - 12xBe^{-6x}$$

$$2dy/dx - 6xy/dx = d^2y/dx^2 - 12xBe^{-6x}$$

$$(2-6x) \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12xBe^{-6x}$$

$$\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12xBe^{-6x}$$

$2-6x$

$$B = \frac{(2-6x) \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12xe^{-6x}}$$

Subs A and B into the general equation

$$y = \left(\frac{6xy/dx + d^2y/dx^2}{-8x^2e^{-6x}} \right) e^{-6x} + \left(\frac{(2-6x) \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12xe^{-6x}} \right) e^{-6x}$$

$$y = \frac{6xy/dx + d^2y/dx^2}{-8x^2} + \frac{(2-6x) \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12x}$$

$$18x \frac{dy}{dx} + 3 \frac{d^2y}{dx^2} + (4x - 12x^2) \frac{dy}{dx} - 2x \frac{d^2y}{dx^2} - 24x^2$$

$$y \left(\frac{18x + 4x - 12x^2}{-24x^2} \frac{dy}{dx} + \frac{3 - 2x}{-24x^2} \frac{d^2y}{dx^2} \right)$$

$$y = \frac{(22x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2}}{-24x^2}$$

$$-24x^2 y = (22x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2}$$

$$(22x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2} + 24x^2 y = 0$$