

## CIVIL ENGINEERING

(2) A differential equation is a relationship between an independent variable  $x$  and a dependent variable  $y$  and one or more derivatives of  $y$  with respect to  $x$

E.g.  $\frac{dy}{dx} = y + yx^{-1}$

b)  $y = Ae^{-4x} + Be^{-6x}$

It's a second order differential equation because it contains two variables

Solution

$$\frac{dy}{dx} = -4x Ae^{-4x} - 6x Be^{-6x} \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} = +16x^2 Ae^{-4x} + 36x^2 Be^{-6x} \quad \text{--- (2)}$$

Solving equation (1) and (2) simultaneously

Multiplying equ (1) by  $6x$  and equ (2) by 1

$$6x \frac{dy}{dx} = -24x^2 Ae^{-4x} - 36x^2 Be^{-6x} \quad \text{--- (3)}$$

$$\frac{d^2y}{dx^2} = 16x^2 Ae^{-4x} + 36x^2 Be^{-6x} \quad \text{--- (4)}$$

$$6x \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8x^2 Ae^{-4x}$$

$$A = \frac{6x \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8x^2 e^{-4x}} \quad \text{--- (5)}$$

Substituting equ (5) into equ (1)

$$\frac{dy}{dx} = -4x \left( \frac{6x \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8x^2 e^{-4x}} \right) e^{-4x} - 6x Be^{-6x}$$

$$\frac{dy}{dx} = \left( \frac{6x \frac{dy}{dx} + \frac{d^2y}{dx^2}}{2x} \right) - 6x Be^{-6x}$$

Multiply through by 2

$$2 \frac{dy}{dx} = 6x \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12x B e^{-6x}$$

$$2 \frac{dy}{dx} - 6x \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12x B e^{-6x}$$

$$(2-6x) \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12x B e^{-6x}$$

$$\frac{dy}{dx} = \frac{\frac{d^2y}{dx^2} - 12x B e^{-6x}}{2-6x}$$

$$B = \frac{(2-6x) \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12x e^{-6x}}$$

Subs A and B into the generated equation

$$y = \left( \frac{6x \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8x^2 e^{-4x}} \right) e^{-4x} + \left[ \frac{2-6x \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12x e^{-6x}} \right] e^{-6x}$$

$$y = \frac{6x \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8x^2} + \frac{(2-6x) \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12x}$$

$$\frac{18x \frac{dy}{dx} + 3 \frac{d^2y}{dx^2} + (4x - 12x^2) \frac{dy}{dx} - 2x \frac{d^2y}{dx^2}}{-24x^2}$$

$$y \frac{(18x + 4x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2}}{-24x^2}$$

$$y = \frac{(22x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2}}{-24x^2}$$

$$-24x^2 y = (22x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2}$$

$$(22x - 12x^2) \frac{dy}{dx} + (3 - 2x) \frac{d^2y}{dx^2} + 24x^2 y = 0$$