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Mechanical Engineering
ENG 282

1a A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x.

Examples: (i) $\frac{dy}{dx} = 2 + y/x$

(ii) $dy/dx = y + y/x$

b $y = Ae^{-4x} + Be^{-6x}$

i A second order differential equation

ii A second order differential equation can be formed because it contains 2 constants in the degenerate equation.

iii $y = Ae^{-4x} + Be^{-6x}$

Solution.

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \text{ ——— (i)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \text{ ——— (ii)}$$

Solving eqn(i) and (ii) simultaneously

multiply eqn (i) by 6

$$\therefore 6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \text{ ——— (iii)}$$

$$\frac{d^2y}{dx^2} = +16Ae^{-4x} - 36Be^{-6x} \text{ ——— (iv)}$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \text{ ——— (v)}$$

Substituting eqn (v) into eqn (i)

$$\frac{dy}{dx} = 4\left(\frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{\partial y}{\partial x} = \frac{6\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}}{2} - 6Be^{-6x}$$

multiply through by 2

$$\therefore \frac{2\partial y}{\partial x} = \frac{6\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2} = 12Be^{-6x}$$

$$\frac{2\partial y}{\partial x} - \frac{6\partial y}{\partial x} = \frac{\partial^2 y}{\partial x^2} - 12Be^{-6x}$$

$$-4\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2} = -12 \cdot Be^{-6x}$$

$$\frac{-4\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2}}{-12e^{-6x}} - B \qquad \therefore \frac{4\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}}{12e^{-6x}} = B$$

Subtitute A and B into the degenerate equation.

$$\therefore y = \frac{6\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}}{-8\cancel{e^{-4x}}} \times \cancel{e^{-4x}} + \frac{4\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}}{12\cancel{e^{-6x}}} \times \cancel{e^{-6x}}$$

$$y = \frac{6\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}}{-8} + \frac{4\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2}}{+12}$$

$$y = \frac{-72\frac{\partial y}{\partial x} - 12\frac{\partial^2 y}{\partial x^2} + 32\frac{\partial y}{\partial x} + 8\frac{\partial^2 y}{\partial x^2}}{96}$$

$$y = \frac{-40\frac{\partial y}{\partial x} - 4\frac{\partial^2 y}{\partial x^2}}{96}$$

$$96y = -40\frac{\partial y}{\partial x} - 4\frac{\partial^2 y}{\partial x^2}$$

$$24y = -10\frac{\partial y}{\partial x} - \frac{\partial^2 y}{\partial x^2}$$

$$\therefore \frac{\partial^2 y}{\partial x^2} + 10\frac{\partial y}{\partial x} + 24y = 0$$