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MAT NO: 17/ENG05/041
DEPT: MECHATRONIC ENGINEERING

Assingment 1

a) Define differential equation and give two examples

A differential equation is a relationship between an independent variable, x a dependent variable, y and one or more derivatives of y with respect to x.

Examples:

i) $xy\frac{dy^2}{dx^2} + y\frac{dy}{dx} + e^{3x} = 0$

ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = \sin 2x$

b) An expression has been obtained for an engineering system to be as given in equation (i)

$$y = Ae^{-4x} + Be^{-6x}$$

i) what is the order of the differential equation that can be formed from the expression?

Second-order equation

ii) Given a reason for your answer in b(i)

It is because a function with 2 - arbitrary Constants gives a 2nd order equation.

iii) form a differential equation from the expression

$$y = Ae^{-4x} + Be^{-6x} \quad \text{---- (1)}$$

$$\frac{dy}{dx} = -4Ae^{-4x} + (-6)Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \ldots\ldots (iii)$$

From equation (ii)

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$$

$$4Ae^{-4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$A = \left(-\frac{dy}{dx} - 6Be^{-6x}\right) \cdot \frac{1}{4e^{-4x}} \quad \ldots\ldots (iv)$$

Substitute eqn (iv) in eqn (iii)

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 16\left(-\frac{dy}{dx} - 6Be^{-6x}\right)e^{-4x} \cdot \frac{1}{4e^{-4x}} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -\frac{4dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -\frac{4dy}{dx} + 12Be^{-6x}$$

$$B = \left(\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right) \cdot \frac{1}{12e^{-6x}} \quad \ldots\ldots (v)$$

Substitute eqn (v) in eqn (iv)

$$A = \left(-\frac{dy}{dx} - 6\left(\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right)e^{-6x} \cdot \frac{1}{12e^{-6x}}\right) \cdot \frac{1}{4e^{-4x}}$$

$$A = \left(-\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right) \cdot \frac{1}{4e^{-4x}} \quad \ldots (vi)$$

$$A = \left(-3\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right) \cdot \frac{1}{4e^{-4x}} \quad \ldots\ldots (vi)$$

Put eqn (vi) and eqn(v) in eqn (1)

$$y = Ae^{-4x} + Be^{-6x} \quad \ldots (1)$$

$$y = \left(-3\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right) e^{-4x} \cdot \frac{1}{4e^{-4x}} + \left(\frac{d^2y}{dx} + 4\frac{dy}{dx}\right) \cdot e^{-6x} \cdot \frac{1}{12e^{-6x}}$$

$$y = \frac{-3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{12}\frac{d^2y}{dx} + \frac{1}{3}\frac{dy}{dx}$$

$$y = \frac{-5}{12}\frac{dy}{dx} - \frac{1}{24}\frac{d^2y}{dx^2}$$

$$y = \left(-5\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right) \cdot \frac{1}{12}$$