Ibrahim Abdul-Hamid
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Mechanical Engineering

1(a) A differential equation is a relationship between an independent Variable (x) and dependent Variable (y) and one or more derivative of y with respect to the x.

Examples (i): $dy/dx = 2 + y/x$

(ii): $dy/dx = y + y/x$

(b) $y = Ae^{-4x} + Be^{-6x}$

(i) A second order differential equation

(ii) A second order differential equation can be formed because it contains 2 constants in the degenerate equation.

(iii) $y = Ae^{-4x} + Be^{-6x}$

Sol.

$$dy/dx = -4Ae^{-4x} - 6Be^{-6x} \quad \cdots (1)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \cdots (2)$$

Solving equation (1) and (2) Simultaneously multiply equation (1) by (6)

$$\therefore 6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \cdots (3)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \cdots (4)$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} \quad \cdots (5)$$

Substituting equ (5) into equ (1)

$$\frac{dy}{dx} = 4\left(\frac{6\,dy/dx + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\,dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

Multiply through by 2

$$2\frac{dy}{dx} = 6\,dy/dx + d^2y/dx^2 - 12Be^{-6x}$$

$$2dy - 6dy = d^2y - 12Be^{-6x}$$

$$\frac{4\,dy/dx - d^2y/dx^2}{-12Be^{-6x}} = B \quad \therefore \quad \frac{4\,dy/dx + d^2y/dx^2}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equat

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8\cancel{e^{-4x}}} \times \cancel{e^{-4x}} + \frac{4\,dy/dx + d^2y/dx^2}{12\cancel{e^{-4x}}} \times \cancel{e^{-4x}}$$

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8} + \frac{4\,dy/dx + d^2y/dx^2}{12}$$

$$y = \frac{-40\,dy/dx - 4\,d^2y/dx^2}{96}$$

$$96y = -40\,dy/dx - 4\,d^2y/dx^2$$

$$24y = -10\,dy/dx - d^2y/dx^2$$

$$\therefore \quad \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$