

1a. A differential equation is a relationship between an independent variable ( $x$ ) and dependent variable ( $y$ ) and one or more derivatives of  $y$  with respect to  $x$ .

Examples: (i)  $\frac{dy}{dx} = 2 + \frac{y}{x}$

(ii)  $\frac{dy}{dx} = y + \frac{y}{x}$

b.  $y = Ae^{-4x} + Be^{-6x}$

i) A second order differential equation.

ii) A second order differential equation can be formed because it contains 2 constants in the degenerate equation.

iii.  $y = Ae^{-4x} + Be^{-6x}$

Solution.

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (2)}$$

Solving eqn (1) and (2) simultaneously  
multiply eqn (1) by 6

$$\therefore 6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{--- (3)}$$

$$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (4)}$$

$$6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \quad \text{--- (5)}$$

Substituting eqn (5) into eqn (1)

$$\frac{dy}{dx} = 4 \left( \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2

$$\therefore 2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B \quad \therefore \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation.

$$\therefore y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12e^{-6x}}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12}$$

$$y = \frac{-72 \frac{dy}{dx} - 12 \frac{d^2y}{dx^2} + 32 \frac{dy}{dx} + 8 \frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}}{96}$$

$$96y = -40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$