

Question 2.

$$F = x^2 i + (3x+2)j + \sin x k.$$

a) $\frac{dF}{dx}$

b) $\frac{d^2 F}{dx^2}$

c) $\left| \frac{dF}{dx} \right|$

d) $\frac{d}{dx} (F \cdot F)$ at $x=1$.

Solution

a) $\frac{dF}{dx} = 2xi + 3j + \cos x k$

b) $\frac{d^2 F}{dx^2} = 2i - \sin x k$

c) $\left| \frac{dF}{dx} \right|$ at $x=1$.

\therefore where $\frac{dF}{dx} = 2xi + 3j + \cos x k$.

$= 2(1)i + 3j + \cos(1)k$

$= 2i + 3j + 0.99k$

$\therefore \left| \frac{dF}{dx} \right| = \sqrt{(2)^2 + (3)^2 + (0.99)^2}$

$= \sqrt{4 + 9 + 0.98}$

$= \sqrt{13.98}$

$= \sqrt{13.99}$

$= 3.74$

d) $\frac{d}{dx} (F \cdot F)$ at $x=1$

where $F = x^2 i + (3x+2)j + \sin x k$.

$\therefore F \cdot F = (x^2 i + (3x+2)j + \sin x k) \cdot (x^2 i + (3x+2)j + \sin x k)$

$F \cdot F = x^4 + (3x+2)(3x+2) + \sin x \cdot \sin x$ where $i \cdot i = j \cdot j = k \cdot k = 1$

$F \cdot F = x^4 + 9x^2 + 12x + 4 + \sin^2 x$

$\therefore \frac{d(F \cdot F)}{dx} = 4x^3 + (18x + 12 + 2 \sin x \cos x)$

Differentiating $\sin^2 x$.

Using the Product's

$$dy/dx = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\sin^2 x = \sin x \cdot \sin x$$

$$u = \sin x, \quad v = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$\frac{dv}{dx} = \cos x$$

$$\therefore \frac{dy}{dx} = \sin x \cdot \cos x + \sin x \cdot \cos x \\ = 2 \sin x \cos x$$

$$\therefore \frac{d(F \cdot F)}{dF}$$

From

$$F \cdot F = x^4 + 9x^2 + 12x + 4 + \sin^2 x$$

$$\frac{d(F \cdot F)}{dF} = 4x^3 + 18x + 12 + 2 \sin x \cos x$$

$$\text{at } x = 1$$

$$\therefore \frac{d(F \cdot F)}{dF} = 4(1)^3 + 18(1) + 12 + 2 \sin(1) \cos(1)$$

$$= 4 + 18 + 12 + 2(0.84 \times 0.76)$$

$$= 34 + 0.0349$$

$$= 34.0349$$

$$\frac{d(F \cdot F)}{dF} \approx 34.03$$

② Where $r = (t^2 + 3t)i - 2 \sin 3t j + 3e^{2t} k$.

① dr/dt

② d^2r/dt^2

③ $|d^2r/dt^2|$ at $t=0$.

Solution;

① dr/dt

$$r = (t^2 + 3t)i - 2 \sin 3t j + 3e^{2t} k$$

② $dr/dt = (2t + 3)i - 6 \cos 3t j + 6e^{2t} k$

③ $d^2r/dt^2 = 2i + 18 \sin 3t j + 12e^{2t} k$

④ at $t=0$, $d^2r/dt^2 = 2i + 18 \sin(0) j + 12e^{2(0)} k$

$$= 2i + 18 \sin(0) j + 12e^{2(0)} k$$

$$= 2i + 0j + 12k$$

$$\therefore \left| \frac{d^2 r}{dt^2} \right| \text{ at } t=0.$$

$$\approx \sqrt{(2)^2 + (0)^2 + (12)^2}$$

$$\approx \sqrt{4+144}$$

$$\approx \sqrt{148}$$

$$\approx 12.165$$

$$\approx 12.17$$