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16/ENG 06/057
Mechanical Engr.
Eng. 382

Assignment 2

$$f(x) = e^{-0.5x} (4-x) - 2$$

From Newton Raphson's eqn; $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

$$f(x) = e^{-0.5x} (4-x) - 2$$

$$f'(x) = \frac{d}{dx} (e^{-0.5x} (4-x) - 2)$$

; let $u = 4-x$ (Using Product rule).
 $v = e^{-0.5x}$

$$\therefore f'(x) = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$f'(x) = e^{-0.5x} (-1) + (4-x) (-0.5e^{-0.5x}) - 0$$

$$f'(x) = -e^{-0.5x} - 0.5e^{-0.5x} (4-x) - 0$$

$$f'(x) = -0.5e^{-0.5x} (x-4) - e^{-0.5x}$$

Recall $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$; at $i=0$, $x=0.5$.

$$\therefore x_{0+1} = x_0 - \frac{e^{-0.5x_0} (4-x_0) - 2}{-0.5e^{-0.5x_0} (x_0-4) - e^{-0.5x_0}}$$

$$\ast x_1 = 0.5 - \frac{0.7258027407}{-2.141702153} \Rightarrow 0.838890606$$

$$\begin{aligned} \text{at } i=1; x_2 &= x_1 - \frac{e^{-0.5x_1} (4-x_1) - 2}{-0.5e^{-0.5x_1} (x_1-4) - e^{-0.5x_1}} \\ &= 0.8388 - \frac{e^{-0.5(0.8388)} (4-0.8388) - 2}{-0.5e^{-0.5 \times 0.8388} (0.8388-4) - e^{-0.5 \times 0.8388}} \end{aligned}$$

$$\ast x_2 = 0.8849565869$$

$$\begin{aligned} \text{at } i=2; x_3 &= x_2 - \frac{e^{-0.5x_2} (4-x_2) - 2}{-0.5e^{-0.5x_2} (x_2-4) - e^{-0.5x_2}} \\ x_3 &= 0.88493 - \frac{e^{-0.5 \times 0.8849} (4-0.8849) - 2}{-0.5e^{-0.5 \times 0.8849} (0.8849-4) - e^{-0.5 \times 0.8849}} \end{aligned}$$

$$* x_3 = 0.8849565869.$$

$$\text{at } i=3; x_4 = x_3 - \frac{e^{-0.5x_3}(4-x_3) - 2}{0.5e^{-0.5x_3}(x_3-4) - e^{-0.5x_3}}.$$

$$\Rightarrow 0.884956 + \frac{1.235611341 \times 10^{-3}}{1.00519922}$$

$$* x_4 = 0.8861914328.$$

$$\text{at } i=4; x_5 = x_4 - \frac{e^{-0.5x_4}(4-x_4) - 2}{0.5e^{-0.5x_4}(x_4-4) - e^{-0.5x_4}}$$

$$x_5 = 0.8861914328 - \frac{e^{-0.5 \times 0.8862}(4 - 0.8862) - 2}{0.5 \times e^{-0.5 \times 0.8862}(0.8862 - 4) - e^{-0.5 \times 0.8862}}$$

$$* x_5 = 0.88570721.$$

$$\text{at } i=5; x_6 = x_5 - \frac{e^{-0.5x_5}(4-x_5) - 2}{0.5e^{-0.5x_5}(x_5-4) - e^{-0.5x_5}}$$

$$x_6 = 0.885708721 - \frac{e^{-0.5 \times 0.8852}(4 - 0.8852) - 2}{0.5e^{-0.5 \times 0.8852}(0.8852 - 4) - e^{-0.5 \times 0.8852}}$$

$$* x_6 = 0.885708721.$$

$$\sum_{i=0}^n \frac{x_{i+1} - x_i}{x_{i+1}} \times 100$$

$$\text{at } i=0;$$

$$\sum_{i=0}^1 x_1 = \frac{0.838890606 - 0.5 \times 100}{0.838890606}$$

$$\rightarrow 40.3947297\%$$

$$\text{at } i=1;$$

$$\sum_{i=1}^2 x_2 = \frac{0.88495869 - 0.838890606 \times 100}{0.88495869}$$

$$\rightarrow 5.205450939\%$$

$$\text{at } i=2;$$

$$\sum_{i=2}^3 x_3 = \frac{0.8849565869 - 0.88495869 \times 100}{0.8849565869}$$

$$\rightarrow 0.0132226\%$$

$$\text{at } i=3; \quad E_9 x_4 = \left| \frac{x_4 - x_3}{x_4} \right| \times 100$$

$$= \frac{0.8861914328 - 0.8847565869}{0.8861914328} \times 100$$

$$\rightarrow 0.1393430194$$

$$\text{at } i=4; \quad E_9 x_5 = \left| \frac{x_5 - x_4}{x_5} \right| \times 100$$

$$E_9 x_5 = \left| \frac{0.885708721 - 0.8861914328}{0.885708721} \right| \times 100$$

$$\rightarrow 0.05450006176$$

$$\text{at } i=5;$$

$$E_9 x_6 = \left| \frac{x_6 - x_5}{x_6} \right| \times 100$$

$$= \left| \frac{0.885708721 - 0.885708721}{0.885708721} \right| \times 100$$

$$\rightarrow 0$$

i	x_{i+1}	$E_9 x_{(i+1)}$
0	0.8388906	40.39247299
1	0.88475658	5.205450932
2	0.88475658	0.0132226
3	0.886191432	0.1393430194
4	0.885708721	0.05450006176
5	0.885708721	0