Assignment 2

16/ENG01/002 ADEOLU JOHN OLA

Chemical Engineering

Solution

$$f(x) = e^{-0.5x}(4-x) - 2$$

From Newton Raphson's equation

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{-0.5x}(4-x) - 2$$

$$f'(x) = \frac{d(e^{-0.5x}(4-x) - 2)}{dx}$$

Let $u = 4-x$ (Using product rule)

$v = e^{-0.5x}$

$$\therefore f'(x) = v\,du/dx + u\,dv/dx$$

$$f'(x) = e^{-0.5x}(-1) + (4-x)(0.5e^{-0.5x}) - 0$$

$$f'(x) = -e^{-0.5x} - 0.5e^{-0.5x}(4-x) - 0$$

$$f'(x) = 0.5e^{-0.5x}(x-4) - e^{-0.5x}$$

Recall $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

at $i=0$, $x = 0.5$

$$\therefore x_{0+1} = x_0 - \frac{e^{-0.5x_0}(4-x_0) - 2}{0.5e^{-0.5x_0}(x_0-4) - e^{-0.5x_0}}$$

$$x_1 = 0.5 - \frac{e^{-0.5\times0.5}(4-0.5) - 2}{0.5e^{-0.5\times0.5}(0.5-4) - e^{-0.5\times0.5}}$$

$$x_1 = 0.5 - \frac{0.725802648}{-2.147402153}$$

$$= 0.838820606$$

at $i=1$, x

$$x_2 = x_1 - \frac{e^{-0.5x_1}(4 - x_1) - 2}{0.5e^{-0.5x_1}(x_1 - 4) - e^{-0.5x_1}}$$

$$x_2 = 0.8388 - \frac{e^{-0.5 \times 0.8388}(4 - 0.8388) - 2}{0.5e^{-0.5 \times 0.8388}(0.8388 - 4) - e^{-0.5 \times 0.8388}}$$

$$x_2 = 0.838890606 + 0.046059809$$

$$x_2 = 0.884950589$$

ab $i = 2$

$$x_3 = x_2 - \frac{e^{-0.5x_2}(4 - x_2) - 2}{0.5e^{-0.5x_2}(x_2 - 4) - e^{-0.5x_2}}$$

$$x_3 = 0.88495 - \frac{e^{-0.5 \times 0.8847}(4 - 0.88496) - 2}{0.5e^{-0.5 \times 0.88495}(0.88495 - 4) - e^{-0.5 \times 0.88495}}$$

$$x_3 = 0.884956589 - \frac{1.23561137 \times 10^{-3}}{-1.643060092}$$

$$x_3 = 0.884956589 + 0.0037263$$

$$x_3 = 0.884956589$$

ab $i = 3$

$$x_4 = x_3 - \frac{e^{-0.5x_3}(4 - x_3) - 2}{0.5e^{-0.5 \times 0.884956}(0.88495 - 4)}$$

$$= 0.884956589 + \frac{1.23561137 \times 10^{-3}}{1.006199222}$$

$$x_4 = 0.886191432 8$$

at $i = 4$

$$x_5 = x_4 - \frac{e^{-0.5x_4}(4 - x_4) - 2}{0.5e^{-0.5x_4}(x_4 - 4) - e^{-0.5x_4}}$$

$$x_5 = 0.8861914328 - \frac{e^{-0.5 \times 0.8862}(4 - 0.8862) - 2}{0.5 \times e^{-0.5 \times 0.8862}(0.8862 - 4) - e^{-0.5 \times 0.8862}}$$

$$x_5 = 0.8861914328 - \frac{-7.924738176 \times 10^{-4}}{-1.641679528}$$

$$x_5 = 0.885708721$$

at $i = 5$

$$x_6 = \frac{x_5 - e^{-0.5x_5}(4 - x_5) - 2}{0.5e^{-0.5x_5}(x_5 - 4) - e^{-0.5x_5}}$$

$$x_6 = 0.885708721 - \frac{e^{-0.5 \times 0.8857}(4 - 0.8857) - 2}{0.5e^{-0.5 \times 0.8857}(0.8857 - 4) - e^{-0.5 \times 0.8857}}$$

$$x_6 = 0.885708721 - \frac{1.330267708 \times 10^{-7}}{-0.9999 - 0.642201}$$

$$= 0.885708721$$

$$\varepsilon_{a_{x_{i+1}}} = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

at $i = 0$

$$\varepsilon_{a x_1} = \frac{0.838890606 - 0.5}{0.838890606} \times 100$$

$$= 40.397477299$$

at $i = 1$

$$\varepsilon_{a x_2} = \frac{0.884956586 9 - 0.838890606}{0.884956586 9} \times 100$$

$$= 5.205450932$$

at $i = 2$

$$\varepsilon_{a x_3} = \left| \frac{x_3 - x_2}{x_3} \right| \times 100$$

$$= \frac{0.884956586 9 - 0.884956586 9}{0.884956586 9} \times 100$$

$$= 0.013222 6$$

at $i = 3$

$$\varepsilon_{a x_4} = \left| \frac{x_4 - x_3}{x_4} \right| \times 100$$

$$= \frac{0.886191432 8 - 0.884956586 9}{0.886191432 8}$$

$$= 0.139343019 4$$

at $i = 4$

$$\varepsilon_4 x_3 = \left| \frac{x_3 - x_4}{x_3} \right| \times 100$$

$$\varepsilon_4 x_3 = \left| \frac{0.88570872 - 0.88617432}{0.88570872} \right| \times 100$$

$$= \pm 0.0545000617 6$$

at $i = 5$

$$\varepsilon_4 x_4 = \left| \frac{x_4 - x_5}{x_4} \right| \times 100$$

$$= \left| \frac{0.88570872 - 0.88570872}{0.88570872} \right| \times 100$$

$$= 0 \%$$

Assignment 3

ADEOLU JOHN OLA
16/ENG01/002
Chemical Engineering

Solution

Estimate the value of temperature using Gauss elimination method

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$
$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$
$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$
$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$
$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$
$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -24$$

In matrix form

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 1 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -24 \end{bmatrix}$$

Using Gauss elimination method

Pivot row = row 1, pivot element first element in row

$\therefore f_1 = 2/1$, $f_2 = 1/1$, $f_3 = 5/1$ $f_4 = -3/1$

$f_5 = 4/1$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2-(2(1)) & -1-(2(1)) & 1-(2(-2)) & 1-(2(1)) & 1-(2(3)) & -3-(2(-1)) \\ 1-(1(1)) & 3-(1(1)) & -3-(1(-2)) & -1-(1(1)) & 2-(1(3)) & 1-(1(-1)) \\ 5-(5(1)) & 2-(5(1)) & -1-(5(-2)) & -1-(5(1)) & 2-(5(3)) & 1-(5(-1)) \\ -3-(-3(1)) & -1-(-3(1)) & 2-(-3(-2)) & 3-(-3(1)) & 1-(-3(3)) & 3-(-3(-1)) \\ 4-(4(1)) & 3-(4(1)) & 1-(4(-2)) & -6-(4(1)) & -3-(4(3)) & -2-(4(-1)) \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 20-(2\times4) \\ -15-(1\times4) \\ -3-(5\times4) \\ 11-(-3\times4) \\ -27-(4\times4) \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & -10 & -15 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

Pivot row is row 2,

Pivot element is second element in row 2

$f_1 = +2/3$, $f_2 = -3/-3 = 1$, $f_3 = -2/3$, $f_4 = -1/-3 = 1/3$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & +5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2-(0(-2/3)) & -1-(-2/3(-5)) & 2-(-1(-2/3)) \\ 0 & 0 & 9-(1(5)) & -6-(0(1)) & -13-(-5(1)) & 6-(-1(1)) \\ 0 & 0 & -4-(5(-2/3)) & 6-(0(-2/3)) & 10-(-5(-2/3)) & 0-(-1(-2/3)) \\ 0 & 0 & 9-(5(1/3)) & -10-(0(1/3)) & -15-(1(-5)) & 2-(-1(1/3)) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \\ -19+(\frac{7}{3}\times12) \\ -23-(12) \\ 28+\frac{7}{3}(12) \\ -43-\frac{1}{3}(12) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -2/3 & 6 & 20/3 & -2/3 \\ 0 & 0 & 22/3 & -10 & -70/3 & 7/3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ 40 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & 5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4 - 12/7(7/3) & -6 - (12/7(-2)) & -8 - \left(\frac{12}{7}\left(-\frac{13}{3}\right)\right) & 7 - \frac{12}{7}\left(\frac{4}{3}\right) \\ 0 & 0 & -2/3 + 2/7(11/3) & 6 + (2/7(-2)) & \frac{20}{3} + \frac{2}{7}\left(-\frac{13}{3}\right) & -2/3 + 2/7\left(\frac{4}{3}\right) \\ 0 & 0 & 22/3 - 22/7(7/3) & -10 - (22/7(-2)) & -70/3 - \frac{22}{7}\left(-\frac{13}{3}\right) & \frac{7}{3} - \frac{22}{7}\left(\frac{4}{3}\right) \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 - 12/7(-11) \\ 36 + 2/7(-11) \\ 40 - 22/7(-11) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -10/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 30/7 & 38/7 & 24/7 \\ 0 & 0 & 0 & -26/7 & -26/7 & -13/7 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -103/7 \\ 930/7 \\ -87/7 \end{bmatrix}$$

Pivot row is row 4.

Pivot element is fourth element on row 4

$f_1 = 38/7 \div -19/7 = -38/18 = -19/9$

$f_2 = -26/7 \div -19/7 = 26/18 = 13/9$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -15/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 38/7 + \left(\frac{9}{7} \times -\frac{15}{7}\right) & 38/7 - \left(\frac{13}{7} \times -\frac{4}{7}\right) & -2/7 + \left(\frac{13}{7} \times \frac{33}{7}\right) \\ 0 & 0 & 0 & -24/7 + \left(\frac{15}{9} \times \frac{15}{7}\right) & 2/7 - \left(\frac{13}{9} \times -\frac{4}{7}\right) & 13/7 - \left(\frac{10}{7} \times \frac{33}{7}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ \frac{250}{7} - \left(\frac{-15}{7} \times \frac{38}{9}\right) \\ \frac{-10}{7} - \left(\frac{-113}{7} \times \frac{10}{9}\right) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -15/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 & -24/3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 95/9 \end{bmatrix}$$

Pivot row is row 5.

Pivot element is fifth element in row 5

$f_1 = 10/9 \div 38/9 = 10/38$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -15/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 - \left(\frac{10}{38} \times \frac{38}{9}\right) & -24/3 - \left(+\frac{10}{38} \times \frac{29}{3}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 95/9 - \left(\frac{-10}{38} \times \frac{11}{9}\right) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -15/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -213/19 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 215/19 \end{bmatrix}$$

1

Multiplying the matrix $(-213/19)\ T_6 = 213/19$

$$\frac{19}{213} \times -\frac{213}{19} T_6 = \frac{213}{19} \times \frac{19}{213}$$

$$-T_6 = 1$$

$$T_6 = -1$$

Also, $0 + 38/9 T_5 + 29/3 T_6 = -11/9$

at $T_6 = -1$

$$38/9 T_5 = -11/9 + 29/3$$

$$T_5 = 76/9 \times 9/38$$

$$T_5 = 2$$

$$-18/7 T_4 - 4/7 T_5 + 33/7 T_6 = -113/7$$

where $T_5 = 2$, $T_6 = -1$

Therefore, $-18/7 T_4 - 4/7(2) + 33/7(-1) = -113/7$

$$-18/7 T_4 = -113/7 + 41/7$$

$$T_4 = -72 \times \frac{7}{18}$$

$$T_4 = 4$$

To get T_3

$$7/3 T_3 - 2T_4 - 13/3 T_5 + 4/3 T_6 = -11$$

$$-7/3 T_3 - 18 = -11$$

$$7/3 T_3 = -11 + 18$$

$$T_3 = 7 \times \frac{3}{7}$$

$$T_3 = 3$$

To get T_2

$$-3T_2 + 5T_3 - 5T_5 - T_6 = 12$$
$$-3T_2 + 5(3) + 0 - 5(0) + 1 = 12$$
$$-3T_2 + 16 = 12$$
$$-3T_2 = 12 - 16$$
$$T_2 = -2 //$$

To get T_4

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$
$$T_1 - 2 - 2(3) + 4 + 3(2) - (-1) = 4$$
$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$
$$T_1 = 4 - 3$$
$$T_1 = 1$$

Therefore,

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ -1 \end{bmatrix} //$$