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17/ENG03/040
CIVIL ENGINEERING
ENG 282

1a. A differential equation is a relationship between an Independent variable (x) and one or more dependent variable (y) and one or more derivative of y with respect to x

Examples:-

(i) $dy/dx = 4 + y/x$

(ii) $dy/dx = y + y/x$

b. $y = Ae^{-4x} + Be^{-6x}$

(i) A second order differential equation

(ii) A second order differential equation can be formed ~~between~~ because it Contains 2 Constants in the general equation

(iii) $y = Ae^{-4x} + Be^{-6x}$

Solution

$dy/dx = -4Ae^{-4x} - 6Be^{-6x}$ ------(1)

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$ ------(2)

Solving eqn (i) and (ii) Simultaneously multiply eqn (i) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{-----(iii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{----(iv)}$$

eqn (1) + (2)

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \quad \text{-----(v)}$$

Subst eqn (v) into eqn (i)

$$\frac{dy}{dx} = 4\left(\frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$= \frac{6\,dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

$$2\frac{dy}{dx} = 6\,dy/dx + d^2y/dx^2 - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\,dy/dx - d^2y/dx^2}{-12e^{-6x}} = B \qquad \therefore \frac{4\,dy/dx + d^2y/dx^2}{12e^{-6x}} = B$$

Subst A and B into the degenerate equation

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \times e^{-4x} + \frac{4\,\partial y/\partial x + \partial^2 y/\partial x^2}{12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\,\partial y/\partial x + \partial^2 y/\partial x^2}{-8} + \frac{4\,\partial y/\partial x + \partial^2 y/\partial x^2}{12}$$

$$y = \frac{-72\,dy/dx - 12\,d^2y/dx^2 + 32\,dy/dx + 8\,\partial^2 y/\partial x^2}{96}$$

$$y = \frac{-40\,dy/dx - 4\,\partial^2 y/\partial x^2}{96}$$

$$96y = -40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}$$

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{\partial^2 y}{\partial x^2} + 10\frac{\partial y}{\partial x} + 24y = 0$$