

ENG 282

Name:- Ugoitz - Uche Chukwunye

Matric Number:- ~~191205~~ 17/ENG 04/071

Department:- Electrical Electronics Engineering

Assignment I

a) Define a differential equation and give two examples.

A differential equation is a relationship between an independent variable, a dependent variable, y , and one or more derivatives of y with respect to x .

Example:-

$$(i) \quad x y \frac{dy^2}{dx^2} + y \frac{dy}{dx} + e^{3x} = 0$$

$$(ii) \quad \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 10y = \sin 2x$$

b. An expression has been obtained for an engineering system to be given in equation (i)

$$y = Ae^{-4x} + Be^{-bx} \quad \text{--- (1)}$$

(i) What is the order of the differential equation that can be formed from the expression?

Second-order equation

(ii) Give a reason for your answer in (b)(i)

It is because with 2 arbitrary constant gives a 2nd order equation

(iii) Form a differential equation from the expression.

$$y = Ae^{-4x} + Be^{-bx} \quad \text{--- (1)}$$

$$\frac{dy}{dx} = -4Ae^{-4x} - bBe^{-bx} \quad \text{--- (2)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 3bBe^{-bx} \quad \text{--- (3)}$$

from equation (2)

$$\frac{dy}{dx} = -4Ae^{-4x} - bBe^{-bx}$$

$$4Ae^{-4x} = \frac{-dy}{dx} - 6Be^{-bx}$$

$$A = \left(\frac{-dy}{dx} - 6Be^{-bx} \right) \cdot \frac{1}{4e^{-4x}} \quad \text{--- (iv)}$$

Substitute eqn (iv) in eqn (iii)

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-bx}$$

$$\frac{d^2y}{dx^2} = 16 \left(\frac{-dy}{dx} - 6Be^{-bx} \right) \frac{1}{4e^{-4x}} + 36Be^{-bx}$$

$$\frac{d^2y}{dx^2} = -4 \frac{dy}{dx} - 24Be^{-bx} + 36Be^{-bx}$$

$$\frac{d^2y}{dx^2} = -4 \frac{dy}{dx} + 12Be^{-bx}$$

$$B = \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right) \cdot \frac{1}{12e^{-bx}} \quad \text{--- (v)}$$

Substitute eqn (v) in eqn (iv)

$$A = \left(\frac{-dy}{dx} - 6 \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right) \frac{1}{12e^{-bx}} \right) \cdot \frac{1}{4e^{-4x}}$$

$$A = \left(\frac{-dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} \right) \cdot \frac{1}{4e^{-4x}}$$

$$A = \left(-3 \frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} \right) \cdot \frac{1}{4e^{-4x}} \quad \text{--- (vi)}$$

put eqn (vi) and eqn (v) in eqn (i)

$$y = Ae^{-4x} + Be^{-bx} \quad \text{--- (i)}$$

$$y = \left(-3 \frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} \right) \frac{1}{4e^{-4x}} + \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right) \frac{1}{12e^{-bx}}$$

$$y = \frac{-3}{4} \frac{dy}{dx} - \frac{1}{8} \frac{d^2y}{dx^2} + \frac{1}{12} \frac{d^2y}{dx^2} + \frac{1}{3} \frac{dy}{dx}$$

$$y = \frac{-5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2}$$

$$y = \left(-\frac{5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2} \right) \cdot \frac{1}{12}$$