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17/ENG06/040
Mechanical Engineering

1. Given that $F = x^2 i + (3x+2)j + \sin x K$; find
(a) dF/dx (b) $\frac{d^2F}{dx^2}$ (c) $\left|\frac{dF}{dx}\right|$ (d) $d/dx(F \cdot F)$ at $x = 1$

Solution

$F = x^2 i + (3x+2)j + \sin x K$

(a) $\frac{dF}{dx} = 2xi + (3+2)j + \cos x K$

$\therefore \frac{dF}{dx} = 2xi + 3j + \cos x K$

(b) $\frac{d^2F}{dx^2} = 2i - \sin x K$

(c) $\left|\frac{dF}{dx}\right| = \sqrt{(2x)^2 + (3)^2 + (\cos x)^2}$
$= \sqrt{4x^2 + 9 + \cos^2 x}$

At $x = 1$
$= \sqrt{4(1)^2 + 9 + \cos^2(1)}$
$= \sqrt{4 + 9 + \cos^2(1)}$
$= 3.74$

(d) $\frac{d}{dx}(F \cdot F) = (F \cdot F) = [(x^2 i) + (3x+2)j + \sin x K] \cdot [(x^2)i + (3x+2)j + \sin x K]$
$= x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$
$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$
$\therefore \frac{d}{dF}(F \cdot F) = 4x^3 + 18x + 12 + 2\sin x \cos x$

$\frac{d}{dx}(F \cdot F) = 4(1)^3 + 18(1) + 12 + 2\sin(1)\cos(1)$ at $x = 1$
$= 4 + 18 + 12 + 2(0.0175)(\cancel{0.0999}\ 0.999)$
$= 4 + 18 + 12 + 0.035$
$= 34.035$

(2) If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} K$. Determine
(a) dr/dt (b) d^2r/dt^2 (c) $|d^2r/dt^2|$ at $t = 0$

Solution

(a) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t} K$

b) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k$

c) $\left|\frac{d^2r}{dt^2}\right| \therefore 2i + [18\sin(3)(0)]j + 12e^{2(0)}k$

$= 2i + 18\sin 0\, j + 12e^{0}k$

$= 2i + 18(0)j + 12(1)k$

$= 2i + 12k$

$\therefore \left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{4 + 144}$

$= \sqrt{148}$

$= 12.17$