

CIVIL ENGINEERING

- (1) Given that $F = x^2i + (3x+2)j + \sin x k$; Find
 (a) df/dx (b) d^2f/dx^2 (c) $|df/dx|$ (d) $d/dx (F \cdot F)$ at $x=1$

Soln:

$$F = x^2i + (3x+2)j + \sin x k$$

$$(a) \frac{dF}{dx} = 2xi + (3+0)j + \cos x k$$

$$\therefore \frac{dF}{dx} = 2xi + 3j + \cos x k$$

$$(b) \frac{d^2F}{dx^2} = 2i - \sin x k$$

$$(c) \left| \frac{dF}{dx} \right| = \frac{\sqrt{(2x)^2 + (3)^2 + (\cos x)^2}}{\sqrt{4x^2 + 9 + \cos^2 x}}$$

At $x=1$

$$= \sqrt{4(1)^2 + 9 + \cos^2(1)}$$

$$= 3.74$$

$$(d) \frac{d}{dx} (F \cdot F) = (F \cdot F) = [(x^2i) + (3x+2)j + \sin x k] \cdot [(x^2i) + (3x+2)j + \sin x k]$$

$$= 2x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$$

$$= 2x^4 + 9x^2 + 12x + 4 + \sin^2 x$$

$$\frac{d}{dx} (F \cdot F) = 4x^3 + 18x + 12 + 2 \sin x \cos x$$

$$\frac{d}{dx} (F \cdot F) \text{ at } x=1 = 4(1)^3 + 18(1) + 12 + 2 \sin(1) \cos(1)$$

$$= 4 + 18 + 12 + 2(0.0175)(0.999)$$

$$= 4 + 18 + 12 + 0.035$$

$$= 34.035$$

2) If $r = (t^2 + 3t)\mathbf{i} - 2\sin 3t\mathbf{j} + 3e^{2t}\mathbf{k}$. Determine

(a) $\frac{dr}{dt}$ (b) $\frac{d^2r}{dt^2}$ (c) $\left| \frac{d^2r}{dt^2} \right|$ at $t=0$

Soln:

(a) $\frac{dr}{dt} = (2t + 3)\mathbf{i} - 6\cos 3t\mathbf{j} + 6e^{2t}\mathbf{k}$

(b) $\frac{d^2r}{dt^2} = 2\mathbf{i} + 18\sin 3t\mathbf{j} + 12e^{2t}\mathbf{k}$

(c) $\left| \frac{d^2r}{dt^2} \right| = 2\mathbf{i} + [18\sin(3)(0)]\mathbf{j} + 12e^{2(0)}\mathbf{k}$

At $t=0$ $= 2\mathbf{i} + 18\sin 0\mathbf{j} + 12e^0\mathbf{k}$

$$= 2\mathbf{i} + 18(0)\mathbf{j} + 12(1)\mathbf{k}$$

$$= 2\mathbf{i} + 12\mathbf{k}$$

$$\therefore \left| \frac{d^2r}{dt^2} \right| = \sqrt{(2)^2 + (12)^2}$$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{4 + 144}$$

$$= \sqrt{148}$$

$$= 12.17$$