

b) A differential equation is a relationship between an independent variable 'x' and dependent variable 'y' and one or more derivatives of y with respect to x.
Eg (i) $\frac{dy}{dx} y + \frac{y}{x}$

b) $y = Ae^{-4x} + Be^{-6x}$

i) A second order differential equation.

ii) This is because it contains two variables.

iii) $y = Ae^{-4x} + Be^{-6x}$.

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \dots (1)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \dots (2)$$

Solve eqn (1) and (2) simultaneously.

Multiply eqn (1) by 6.

$$6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \dots (3)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \dots (4)$$

$$6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

Subst. eqn (5) into eqn (1).

$$\frac{dy}{dx} = 4 \left(\frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2.

$$2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12B e^{-6x}.$$

$$\frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B.$$

$$\therefore B = \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

Sub. A and B into the degenerate equation.

$$\therefore y = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \times e^{-4x} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \times e^{-6x}}{12e^{-6x}}$$

$$y = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}.$$

$$y = 18 \frac{dy}{dx} + 3 \frac{d^2y}{dx^2} - 8 \frac{dy}{dx} - 2 \frac{d^2y}{dx^2}$$

$$y = 10 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$-24y = 10 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0.$$