

1 ENG 282 Assignment 2

Given that

$$F = x^2 i + (3x+2)j + \sin x k$$

a. $\frac{dF}{dx} = 2xi + 3j + \cos x k$

b. $\frac{d^2 F}{dx^2} = 2i - \sin x k$

c. $\frac{dF}{dx}$ at $x=1$

$$\frac{dF}{dx} = 2i + 3j + 0.9998k$$

$$\left| \frac{dF}{dx} \right| = \sqrt{(2)^2 + (3)^2 + (0.9998)^2}$$

$$= 3.74 \text{ units}$$

d. $\frac{d(F \cdot F)}{dx}$

$$F \cdot F = (x^2 i + (3x+2)j + \sin x k) \cdot (x^2 i + (3x+2)j + \sin x k)$$

$$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$$

Note: $i \cdot i = 1$

$j \cdot j = 1$

$k \cdot k = 1$

$$\therefore \frac{d(F \cdot F)}{dx} = 4x^3 + 18x + 12 + 2 \sin x \cos x$$

at $x=1$

$$\frac{d(F \cdot F)}{dx} = 4(1)^3 + 18(1) + 12 + 2 \sin(1) \cos(1)$$

$$= 4 + 18 + 12 + 0.035$$

$$= 34.035$$

Put equation (IV) in equation (II) multiply by 4

$$d^2y/dx^2 = 16 \left[\frac{dy}{dx} + 60 \right] x - 1 + 365$$

$$\frac{d^2y}{dx^2} = -4 \frac{dy}{dx} - 245 + 365$$

$$\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} = 120$$

$$D^2 = \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right) \frac{1}{12} \dots \text{--- (V)}$$

Put equation (V) in equation (IV)

$$R = \left[\frac{dy}{dx} + 6 \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right) \frac{1}{12} \right] x - \frac{1}{4}$$

$$R = \left(\frac{dy}{dx} + \frac{d^2y}{2 dx^2} + 2 \frac{dy}{dx} \right) x - \frac{1}{4}$$

$$R = \left[3 \frac{dy}{dx} + \frac{1}{2} \frac{d^2y}{dx^2} \right] x - \frac{1}{4} \dots \text{--- (VI)}$$

Put equation (VI) and (I) in equation (I)

$$y = \left(\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} \right) \frac{1}{12} + \left[\left(3 \frac{dy}{dx} + \frac{1}{2} \frac{d^2y}{dx^2} \right) x - \frac{1}{4} \right]$$

$$y = \frac{1}{12} \frac{d^2y}{dx^2} + \frac{1}{3} \frac{dy}{dx} - \frac{3}{4} \frac{dy}{dx} - \frac{1}{4} \frac{d^2y}{dx^2}$$

$$y = \frac{-1}{24} \frac{d^2y}{dx^2} - \frac{5}{12} \frac{dy}{dx}$$

$$y + \frac{1}{24} \frac{d^2y}{dx^2} + \frac{5}{12} \frac{dy}{dx} = 0$$

Multiply through by 24

$$24y + \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} = 0$$