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17/mth01/083
Civil Engineering

A A differential equation is the relationship between an independent variable x, a dependent variable y and one or more derivatives of y with respect to x.

e.g. $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} + e^{3x} = 0$

$= x^2 dy/dx = y \cdot \sin x = 0$

B An expression has been obtained for an engineering system to be given as in equation 1

$$y = Ae^{-4x} + Be^{-6x}$$

i What is the order of the differential equation that can be formed from the expression?

= second order equation

ii Give reason for your answer in b(i)

= A function with no arbitrary constants gives a second order equation.

iii. From the differential equation from the expression

$$y = Ae^{-4x} + Be^{-6x}$$

$dy/dx = -4Ae^{-4x} + (-6Be^{-6x}) = -4Ae^{-4x} - 6Be^{-6x}$ ---- ii

$d^2y/dx^2 = 16Ae^{-4x} + 36Be^{-6x}$ ---- iii

From equation (ii)

$dy/dx = -4Ae^{-4x} - 6Be^{-6x}$

$4Ae^{-4x} = \frac{-dy}{dx} - 6Be^{-6x}$

$$Ae^{-4x} = \left[-\frac{dy}{dx} - 6Be^{-6x}\right]^{1/4}$$

$$\therefore A = \left[-\frac{dy}{dx} - 6Be^{-6x}\right]^{1/4} e^{-4x} \quad \text{---(iv)}$$

Put equation (iv) into equation (iii)

$$\frac{d^2y}{dx^2} = 16 \times \left(-\frac{dy}{dx} - 6Be^{-6x}\right)^{1/4} e^{-4x} \times e^{4x} + 36Be^{-6x}$$

$$= 4\left[-\frac{dy}{dx} - 6Be^{-6x}\right] + 36Be^{-6x}$$

$$= -\frac{4dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx} = -\frac{4dy}{dx} + 12Be^{-6x}$$

$$B = \left[\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right] \frac{1}{12e^{-6x}} \quad \text{---(v)}$$

Put equation (v) into equation (iv)

$$A = \left[-\frac{dy}{dx} - 6\left(\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right)\frac{1}{12e^{-6x}} \times e^{-6x}\right]^{1/4} e^{-4x}$$

$$A = \left[-\frac{dy}{dx} - \frac{1}{2}\frac{dy^2}{dx^2}\right]^{1/4} e^{-4x} \quad \text{---(vi)}$$

Put equation (vi) into equation (iii)

$$y = \left(-\frac{3dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right)^{1/4} e^{-4x} \times e^{-4x} + \left[\frac{d^2y}{dx^2} + \frac{4dy}{dx}\right] \cdot \frac{1}{12}e^{-8} \times e^{-8}$$

$$y = -\frac{3}{4}\frac{dy}{dx} - \frac{1}{8}\frac{d^2y}{dx^2} + \frac{1}{12}\frac{d^2y}{dx^2} + \frac{1}{3}\frac{dy}{dx}$$

$$y = -\frac{5}{12}\frac{dy}{dx} - \frac{1}{24}\frac{d^2y}{dx^2}$$

$$y = \left[-5\frac{dy}{dx} - \frac{1}{2}\frac{d^2y}{dx^2}\right]\frac{1}{12}$$