

Assignment 2

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Elect / Elect Engg

ELG 382 - Engineering Maths IV

Soln

$$f(x) = e^{-0.5x} [4-x] - 2$$

From Newton Raphson's equation

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x) = e^{-0.5x} (4-x) - 2$$

$$f'(x) = \frac{d}{dx} [e^{-0.5x} (4-x) - 2]$$

Let $u = 4-x$ — Using product rule

$$v = e^{-0.5x}$$

$$f'(x) = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$f'(x) = e^{-0.5x} [-1] + [4-x] [-0.5e^{-0.5x}] - 0 =$$

$$f'(x) = -e^{-0.5x} - 0.5e^{-0.5x} [4-x] - 0$$

$$f'(x) = -0.5e^{-0.5x} [x-4] - e^{-0.5x}$$

$$\text{Recall } x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

@ $i = 0, x = 0.5$

$$x_{0+1} = x_0 - \frac{e^{-0.5x_0} [4-x_0] - 2}{0.5e^{-0.5x_0} [x_0-4] - e^{-0.5x_0}}$$

$$x_1 = 0.5 - \frac{e^{-0.5 \times 0.5} [4-0.5] - 2}{0.5e^{-0.5 \times 0.5} [0.5-4] - e^{-0.5 \times 0.5}}$$

$$x_1 = 0.5 - \frac{0.728827407}{-2.141702163}$$

$$= 0.845890606$$

@ $i = 1$

$$x_2 = x_1 - \frac{e^{-0.5x_1} [4-x_1] - 2}{0.5e^{-0.5x_1} [x_1-4] - e^{-0.5x_1}}$$

$$x_2 = 0.838890606 - \frac{e^{-0.5 \times 0.8388} [4 - 0.8388] - 2}{0.5 e^{-0.5 \times 0.8388}}$$

$$0.5 e^{-0.5 \times 0.8388} [0.8388 - 4] - e^{-0.5 \times 0.8388}$$

$$x_2 = 0.838890606 + 0.0460659609$$

$$x_2 = 0.8849565669$$

@ i = 2

$$x_3 = x_2 - \frac{e^{-0.5 x_2} [4 - x_2] - 2}{0.5 e^{-0.5 x_2}}$$

$$0.5 e^{-0.5 x_2} [x_2 - 4] - e^{-0.5 x_2}$$

$$x_3 = 0.8849565669 - \frac{e^{-0.5 \times 0.884956} [4 - 0.884956] - 2}{0.5 e^{-0.5 \times 0.884956}}$$

$$0.5 e^{-0.5 \times 0.884956} [0.884956 - 4] - e^{-0.5 \times 0.884956}$$

$$= 0.8849565669 + 0.0027263$$

$$x_3 = 0.8849565669$$

@ i = 3

$$x_4 = x_3 - \frac{e^{-0.5 x_3} [4 - x_3] - 2}{0.5 e^{-0.5 x_3}}$$

$$0.5 e^{-0.5 x_3} [x_3 - 4] - e^{-0.5 x_3}$$

$$= 0.8849565669 + \frac{e^{-0.5 \times 0.884956} [4 - 0.8849565669] - 2}{0.5 e^{-0.5 \times 0.884956}}$$

$$0.5 e^{-0.5 \times 0.884956} [0.884956 - 4] - e^{-0.5 \times 0.884956}$$

$$x_4 = 0.8849565669 + \frac{1.235611371 \times 10^{-3}}{1.00619922}$$

$$x_4 = 0.8861914328$$

@ i = 4

$$x_5 = x_4 - \frac{e^{-0.5 x_4} [4 - x_4] - 2}{0.5 e^{-0.5 x_4}}$$

$$0.5 e^{-0.5 x_4} [x_4 - 4] - e^{-0.5 x_4}$$

$$x_5 = 0.8861914328 - \frac{e^{-0.5 \times 0.8862} [4 - 0.8862] - 2}{0.5 e^{-0.5 \times 0.8862}}$$

$$0.5 e^{-0.5 \times 0.8862} [0.8862 - 4] - e^{-0.5 \times 0.8862}$$

$$x_6 = 0.8861914328 - \frac{1.924436176 \times 10^{-4}}{-1.641649528}$$

$$x_6 = 0.886708721$$

@ i = 5

$$x_6 = x_5 - \frac{e^{-0.5 x_5} [4 - x_5] - 2}{0.5 e^{-0.5 x_5}}$$

$$0.5 e^{-0.5 x_5} [x_5 - 4] - e^{-0.5 x_5}$$

$$\begin{aligned}
 x_5 &= 0.855708721 - e^{-0.5 \times 0.8557} [4 - 0.8557] - 2 \\
 &\quad 0.5 e^{-0.5 \times 0.8557} [0.8557 - 4] - e^{-0.5 \times 0.8557} \\
 &= 0.855708721 - 1.33026708 \times 10^{-7} \\
 &\quad - 0.9999 - 0.642201 \\
 &= 0.855708721
 \end{aligned}$$

$$f_{a_{i+1}} = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

$$\begin{aligned}
 f_{a_{21}} &= \frac{0.838890606 - 0.8}{0.838890606} \times 100 \\
 &= 40.397
 \end{aligned}$$

@ i = 1

$$\begin{aligned}
 f_{a_{22}} &= \frac{0.8849565869 - 0.838890606}{0.8849565869} \times 100 \\
 &= 5.205
 \end{aligned}$$

@ i = 2

$$\begin{aligned}
 f_{a_{23}} &= \frac{0.8849565869 - 0.8849565869}{0.8849565869} \times 100 \\
 &= 0.0132
 \end{aligned}$$

@ i = 3

$$\begin{aligned}
 f_{a_{24}} &= \frac{0.8861914328 - 0.8849565869}{0.8861914328} \\
 &= 0.1393
 \end{aligned}$$

@ i = 4

$$\begin{aligned}
 f_{a_{25}} &= \frac{0.885708721 - 0.8861914328}{0.885708721} \times 100 \\
 &= 0.0545
 \end{aligned}$$

@ i = 5

$$\begin{aligned}
 f_{a_{26}} &= \frac{0.885708721 - 0.885708721}{0.885708721} \times 100 \\
 &= 0
 \end{aligned}$$