

Garber Pedlura Ryomule

Wlombos, lolo

Biomedical Engineering

(a) A differential equation is a relationship between an independent variable 'x' and dependent variable 'y' and one or more derivatives of y w.r.t x

E.g $\frac{dy}{dx} = y + \frac{y}{x}$

(b) $y = Ae^{-4x} + Be^{-6x}$

A second order differential equation
This is because it contains two variables

$y = Ae^{-4x} + Be^{-6x}$

Solution

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ --- (1)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ --- (2)

Solving eqns (1) and (2) Simultaneously

and multiplying eqn (1) by 6

$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$ --- (3)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ --- (2)

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$

As $6\frac{dy}{dx} + \frac{d^2y}{dx^2}$ --- (5)

Substitute eqn (5) into eqn (2)

$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} e^{-4x} - 6Be^{-6x}$

$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$

Multiply through by 2

$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12 B e^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12 B e^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = B$$

$$B = \frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{12 e^{-6x}}$$

Substitute A and B into the degenerate equation

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8 e^{-4x}} + \frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{12 e^{-6x}} e^{-6x}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{18 \frac{dy}{dx} + 3 \frac{d^2y}{dx^2}}{-24} - \frac{8 \frac{dy}{dx} - 2 \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{10 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-24}$$

$$-24y = 10 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$