Assignment 3

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Solution

Estimate the value of temperature using Gauss elimination method

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$
$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$
$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$
$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$
$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$
$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -24$$

In matrix form

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 1 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -24 \end{bmatrix}$$

Using Gauss elimination method

Pivot row = row 1, pivot element first element in row 1

$\therefore f_1 = 2/1$, $f_2 = 1/1$, $f_3 = 5/1$ $f_4 = -3/1$

$f_5 = 4/1$

$$\left[\begin{array}{c|c|c|c|c|c} 1 & 1 & -2 & 1 & 3 & -1 \\ 2-(2(1)) & -1-(2(1)) & 1-(2(-2)) & 1-(2(1)) & 1-(2(3)) & -3-(2(-1)) \\ 1-(1(1)) & 3-(1(1)) & -3-(1(-2)) & -1-(1(1)) & 2-(1(3)) & 1-(1(-1)) \\ 5-(5(1)) & 2-(5(1)) & -1-(5(-2) & -1-(5(1)) & 2-(5(3)) & 1-(5(-1)) \\ -3-(-3(1)) & -1-(-3(1) & 2-(-3(-2)) & 3-(-3(1)) & 1-(-3(3)) & 3-(-3(-1)) \\ 4-(4(1)) & 3-(4(1)) & 1-(4(-2)) & -6-(4(1)) & -3-(4(3)) & -2-(4(-1)) \end{array}\right] \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

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$$= \begin{bmatrix} 4 \\ 20-(2\times4) \\ -15-(1\times4) \\ -3-(5\times4) \\ 16-(-3\times4) \\ -27-(4\times4) \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & -10 & -15 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

Pivot row is row 2,

Pivot element is second element in row 2

$f_1 = +2/-3$, $f_2 = -3/-3 = 1$, $f_3 = -2/3$, $f_4 = -1/-3 = 1/3$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & +5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2-(0(-2/3)) & -1-(-2/3(-5)) & 2-((-1)(-2/3)) \\ 0 & 0 & 9-(1(5)) & -6-(0(1)) & -13-(-5(1)) & 6-(-1(1)) \\ 0 & 0 & -4-(5(-2/3)) & 6-(0(-2/3)) & 10-(-5(-2/3)) & 0-(-1(-2/3)) \\ 0 & 0 & 9-(5(1/3)) & -10-(0(1/3)) & -15-(1/3\times-5) & 2-(-1(1/3)) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \\ -19+(\frac{2}{3}\times12) \\ -23-(12) \\ 28+\frac{2}{3}(12) \\ -43-\frac{1}{3}(12) \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -2/3 & 6 & 20/3 & -2/3 \\ 0 & 0 & 22/3 & -10 & -20/3 & 7/3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ 48 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & 5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 4-\frac{12}{7}(\frac{7}{3}) & -6-(\frac{12}{7}(-2)) & -8-(\frac{12}{7}(-\frac{13}{3})) & 7-\frac{12}{7}(\frac{4}{3}) \\ 0 & 0 & -\frac{2}{3}+\frac{2}{7}(\frac{7}{3}) & 6+(\frac{2}{7}(-2)) & \frac{20}{3}+\frac{2}{7}(-\frac{13}{3}) & -\frac{2}{3}+\frac{2}{7}(\frac{4}{3}) \\ 0 & 0 & \frac{22}{3}-\frac{22}{7}(\frac{7}{3}) & -10-(\frac{22}{7}(-2)) & -\frac{20}{3}-\frac{22}{7}(-\frac{13}{3}) & \frac{7}{3}-\frac{22}{7}(\frac{4}{3}) \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35-\frac{12}{7}(-11) \\ 36+\frac{2}{7}(-11) \\ 48-\frac{22}{7}(-11) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 38/7 & 38/7 & 2/7 \\ 0 & 0 & 0 & -26/7 & -26/7 & -13/7 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ 230/7 \\ -87/7 \end{bmatrix}$$

Pivot row is row 4.

Pivot element is fourth element on row 4

$f_1 = 38/7 \div -18/7 = -38/18 = -19/9$

$f_2 = -26/7 \div -18/7 = 26/18 = 13/9$

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$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 38/7 + \left(\frac{17}{9} \times -\frac{17}{7}\right) & 38/7 - \left(\frac{17}{9} \times -\frac{4}{7}\right) & -2/7 + \left(\frac{17}{9} \times \frac{33}{7}\right) \\ 0 & 0 & 0 & -28/7 + \left(\frac{13}{9} \times \frac{17}{7}\right) & 2/7 - \left(\frac{13}{9} \times -\frac{4}{7}\right) & 13/7 - \left(\frac{13}{9} \times \frac{33}{7}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ \frac{230}{7} - \left(\frac{-113}{7} \times \frac{17}{9}\right) \\ -\frac{13}{7} - \left(\frac{-113}{7} \times \frac{13}{9}\right) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 & -28/3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 98/9 \end{bmatrix}$$

Pivot row is row 5.

Pivot element is fifth element in row 5

$P_1 = 10/9 \div 38/9 = 10/38$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 - \left(\frac{10}{38} \times \frac{38}{9}\right) & -28/3 - \left(+\frac{10}{38} \times \frac{29}{3}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 98/9 - \left(-\frac{10}{38} \times \frac{11}{9}\right) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -213/19 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 213/19 \end{bmatrix}$$

$$\begin{matrix} 0 & 0 & 0 \\ & & 1 \end{matrix}$$

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Multiplying the matrix

$$0 + (-213/14)\, x_6 = 213/14$$

$$\frac{14}{213} \times -\frac{213}{14}\, x_6 = \frac{213}{14} \times \frac{14}{213}$$

$$-x_6 = 1$$

$$x_6 = -1$$

Also,

$$0 + 38/9\, x_5 + 29/3\, x_6 = -11/9$$

at $x_6 = -1$

$$38/9\, x_5 = -11/9 + 29/3$$

$$x_5 = 76/9 \times 9/38$$

$$x_5 = 2$$

$$-18/7\, x_4 - 4/7\, x_5 + 33/7\, x_6 = -113/7$$

where $x_5 = 2$, $x_6 = -1$

Therefore,

$$-18/7\, x_4 - 4/7(2) + 33/7(-1) = -113/7$$

$$-18/7\, x_4 = -113/7 + 41/7$$

$$x_4 = \frac{-72}{7} \times \frac{7}{18}$$

$$x_4 = 4$$

To get x_3

$$7/3\, x_3 - 2x_4 - 13/3\, x_5 + 4/3\, x_6 = -11$$

$$-7/3\, x_3 - 18 = -11$$

$$7/3\, x_3 = -11 + 18$$

$$x_3 = 7 \times \frac{3}{7}$$

$$x_3 = 3$$

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To get T_2

$$-3T_2 + 5T_3 - 5T_5 - T_6 = 12$$

$$-3T_2 + 5(3) + 0 - 5(2) + 1 = 12$$

$$-3T_2 + 6 = 12$$

$$-3T_2 = 12 - 6$$

$$T_2 = -2$$

To get T_1

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 - 2 - 2(3) + 4 + 3(2) - (-1) = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 - 3$$

$$T_1 = 1$$

Therefore,

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ -1 \end{bmatrix}$$

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