

Egbonode Christopher Echuwon 171619061028 Mechanical Engineering
Eng 282

1a) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect of x

Example: 1) $\frac{dy}{dx} = 2 + \frac{y}{x}$

2) $\frac{dy}{dx} = y + \frac{y}{x}$

b) $y = Ae^{-4x} + Be^{-6x}$

1) A second order differential equation

2) A second order differential equation can be formed because it contains 2 constants in the degenerate equation

3) $y = Ae^{-4x} + Be^{-6x}$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad \text{--- (i)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (ii)}$$

Solving eqn (i) and (ii) simultaneously

multiply eqn (i) by 6

$$6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad \text{--- (iii)}$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad \text{--- (iv)}$$

$$6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \quad \text{--- (v)}$$

Substituting eqn (v) into eqn (i)

$$\frac{dy}{dx} = 4 \left(\frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

multiply through by 2

$$2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$= \frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B \quad \therefore \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12}$$

$$y = \frac{-72 \frac{dy}{dx} - 12 \frac{d^2y}{dx^2} + 32 \frac{dy}{dx} + 8 \frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}}{96}$$

$$96y = -40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$