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ENG 282

1) Given that $f = x^2 i + (3x+2)j + \sin x K$ find (a) dy/dx b) d^2y/dx^2 c) $|dy/dx|$
d) $d/dx\,(f \cdot f)$ at $x = 1$

Solution

$f = x^2 i + (3x+2)j + \sin x K$

a) $dy/dx = 2xi + (3)j + \cos x K$

$\therefore \frac{dy}{dx} = 2xi + 3j + \cos x K$

b) $\frac{d^2f}{dx^2} = 2i - \sin x K$

c) $\left|\frac{dy}{dx}\right| = \sqrt{(2x)^2 + (3)^2 + (\cos x)^2}$

$= \sqrt{4x^2 + 9 + \cos^2 x}$

At $x = 1 \rightarrow \sqrt{4(1)^2 + 9 + \cos^2(1)}$

$= \sqrt{4 + 9 + 0.999} = \sqrt{13.999} = 3.74$

d) $d/dx\,(f \cdot f) \rightarrow (f \cdot f) = [(x^2 i) + (3x+2)j + \sin x K]$

$= x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$

$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$

$\therefore \frac{d}{dx}(f \cdot f) = 4x^3 + 18x + 12 + 2\sin x \cos x$

$\frac{d}{dx}(f \cdot f) \rightarrow 4(1)^3 + 18(1) + 12 + 2\sin(1)\cos(1)$ at $x = 1$

$= 4 + 18 + 12 + 2(0.0175)(0.999)$

$= 4 + 18 + 12 + 0.035$

$= 34.035$

multiply through by 2

$$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$= \frac{-4\frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B \quad \therefore \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \times e^{-4x} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{+12}$$

$$y = \frac{-12\frac{dy}{dx} - 12\frac{d^2y}{dx^2} + 32\frac{dy}{dx} + 8\frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}}{96}$$

$$96y = -40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}$$

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$