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17/Engen1006

Assignment

It is discovered that  $600 \text{ ft}^3/\text{min}$  of fresh air flows into a room containing  $20000 \text{ ft}^3$  of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of  $60 \text{ ft}^3/\text{min}$ . If the room contains no fresh air initially

a) develop a model for the amount of fresh air in the room at any time,  $t$

Answer

Let  $y(t)$  be the amount of air at time  $t$  in ( $\text{ft}^3$ ) in the room

$$\frac{dy}{dt} = \text{Air Inflow rate} - \text{fresh air outflow rate}$$

Air Inflow rate  $\rightarrow 600 \text{ ft}^3/\text{min}$

fresh air outflow  $\rightarrow R/B$ : the amount flowing out of the room is a function of the amount in the room

$$= \frac{600}{20,000} = 0.03 \text{ min}^{-1}$$

20,000

i.e.  $0.03$  of  $y(t)$  is the outflow  $= 0.03y \text{ ft}^3/\text{min}$

Now

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 200,000)$$

Thus the equation is separate and can be solved

$$\frac{dy}{y - 200,000} = -0.03 dt$$

$(y - 200,000)$

Integrate both sides

$$\ln(y - 200,000) = -0.03t + C$$

$$y - 200,000 = e^{-0.03t} \cdot e^C$$

$$y = 200,000 + C e^{-0.03t} \quad \text{Initial Condition} \quad y = 200,000 = e^{-0.03 \cdot 0} \cdot C$$

If  $t = 0$ ,  $y(t) = 0$  since the room contained no fresh air initially

$$y = 200,000 \cdot C e^{-0.03 \cdot 0}$$

$$0 - 200,000 = C$$

$$\therefore C = -200,000 \quad \text{--- (2)}$$

Put a in equation 2

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000(1 - e^{-0.03t}) \quad (x \times x)$$

The equation above is the model for the amount of fresh air in the room

Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100}$$

$$= 0.9$$

$$y = 0.9 \text{ of } 20000 \text{ be } 90\% \text{ of air in the room}$$

$$= 0.9 \times 20000$$

$$= 18000 \text{ ft}^3$$

$$\therefore y = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$t = \frac{-2.303}{-0.03}$$

$$= 76.77$$

$$= 77 \text{ mins}$$

$\therefore$  The air in the room will be 90% fresh at 77 minutes

c) with the aid of matlab, Plot the dynamic response of the amount of fresh air in the room for  $t = 0$  to  $t = 6$  hrs using a step of 5 min

NB:  $t = 6 \text{ hrs} = 6 \times 60 = 360 \text{ minutes}$

Codes

Command window

Clear all

Close all

Sym  $Y, t, k$   
 $y = 20000 (1 - \exp(-0.03 \times t))$

$t = 0 : 5 : 360$

$Y_n = \text{subs}(4)$

Plot  $(t, Y_n)$

x label (Time (min))

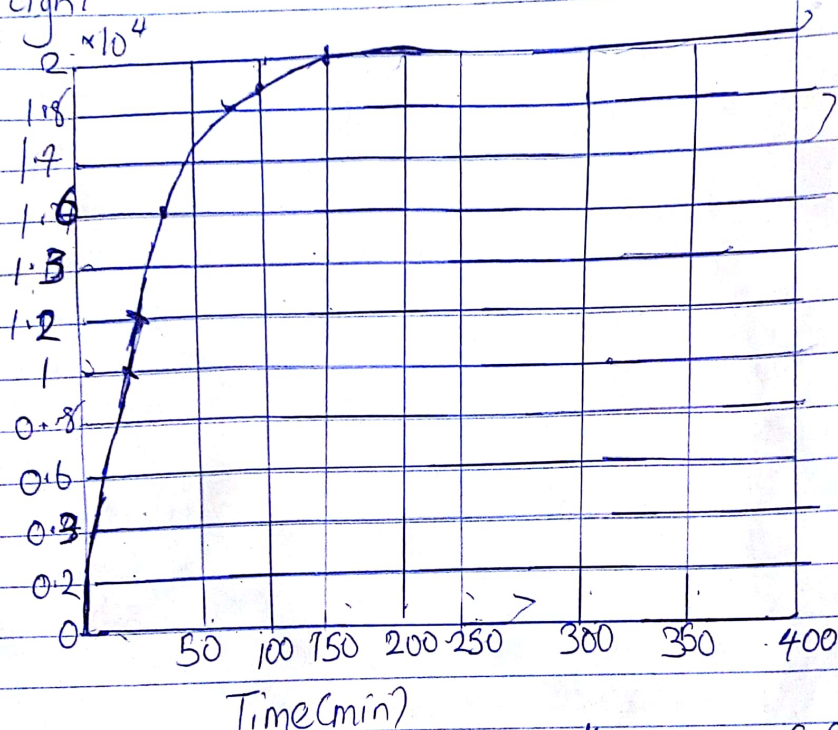
y label (FlowRATE of FRESH AIR  $\text{ft}^3/\text{min}$ )

Grid on

Grid minor

Cons tight

Output  $\times 10^4$



d) Determine the Steady State Value of the amount of fresh air in the room

i) Answer

The steady-state value is  $20000 \text{ft}^3$  at 25mins of exponential approach

e) Comment on answer in d)

The function shows an exponential approach to the limit of  $20000 \text{ft}^3$  as  $q$  increases with time. Also when the steady-state value approach  $20000 \text{ft}^3$  at 45mins and continues till 360 minutes (hours). The model discussed becomes more relative in pneumatic technology, although maybe difficult because mixing may be imperfect