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ENG282 elect/elect 17/ENG04/027

Assignment IV

(1) it is discovered that 600ft³/min of fresh air flows into a room containing 20000ft³ of air. The mixture, which is made practically uniform by circulating fan, is exhausted at the rate of 600 cubic feet per minute (cfm). if the room contains no fresh air initially.

(a) develop a model for the amount of fresh air in the room at any time, t

b) Calculate the time at which 90% of the air in the room will become fresh.

c) with the aid of Matlab, plot the dynamic response of the amount of fresh air in the room for t=0 to t=6 hrs using a step time of 5min

d) determine the steady value of the amount of fresh air in the room, and

e) Comment on the result obtained in (d)

Soln

(a) let y(t) be the amount of air at any time t in ft³ in the room

$\frac{dy}{dt}$ = fresh air inflow rate − fresh air outflow rate

fresh air inflow = 600ft³/min

fresh air outflow NB: the amount flowing out of the room is a function of the amount in the room

Hence $\frac{600}{20000} = 0.03$min

∴ 0.03 of y(t) is the outflow = 0.03yft³/min

Now;

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$\frac{dy}{dt} = 0.03(y - 20000)$

This eqn can be simplified as

$\frac{dy}{(y-20000)} = -0.03dt$

$\ln(y-20000) = -0.03t + C$

$y - 20000 = e^{(-0.03t+C)}$

$y - 20000 = e^{-0.03t} \cdot e^{C}$ let $e^{C} = y_0$

$y - 20000 = e^{-0.03t} \cdot y_0$ ——— (1)

At t=0, y(t)=0 as the room contained no fresh air initially

Hence eqn ① becomes

$y - 20000 = e^{-(0.03t)} \cdot y_0$

$0 - 20000 = e^0 \cdot y_0 \Rightarrow 0 - 20000 = 1 \cdot y_0$

$y_0 = 20000$

Hence Put $y_0 = 20000$ into eqn ①

$\therefore y = 20000 - 20000 e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$

$y = 20000(1 - e^{-0.03t})$ ——— model for the amount of fresh air in the room.

(b) Calculate t

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20,000$

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$\therefore 18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1 \Rightarrow \frac{-0.03t}{-0.03} = \frac{\ln(0.1)}{-0.03}$

$t = 76.77 mins$

(c) $t = 6hrs$

$= 6 \times 60s$

$= 360 mins$

Q6

Command Window

```
Clear
clc
Close all
Syms y,t
t=0:5:360
y=20000*(1-exp(-0.038*t))
yp=subs(y)
Plot(t,yp)
xlabel('time(t)')
ylabel('flow rate of fresh air (ft^3/min)')
grid on
grid minor
```

(d) The steady state value is $2 \times 10^4 ft^3$.

(e) The function y often is an exponential growth and when the steady state value approaches 2×10^4 at 150 mins and continues