

1

a) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivative of y with respect to x .

Examples: i) $\frac{dy}{dx} = 2 + \frac{y}{x}$

ii) $\frac{dy}{dx} = y + \frac{y}{x}$

b) $y = Ae^{-4x} + Be^{-6x}$

c) A Second Order differential equation

ii) A Second Order differential equation can be formed because it contains 2 constants in the degenerate equation.

iii) $y = Ae^{-4x} + Be^{-6x}$

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$ — (1) Solution x

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$ — (2)

Solving eqn (1) and (2) Simultaneously

Multiply eqn (1) by 6

$6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x}$ — (3)

$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-6x}$ — (4)

$6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$

$$A = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \quad (5)$$

Substituting eqn (5) into eqn (4)

$$\frac{dy}{dx} = \frac{4 \left(\frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}}$$

$$\frac{dy}{dx} = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2

$$2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4 \frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B$$

$$\frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$\therefore y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \times e^{4x} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{-12 \frac{dy}{dx} - 12 \frac{d^2y}{dx^2} + 32 \frac{dy}{dx} + 8 \frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-10 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}}{96}$$

$$96y = -10 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$