1. Given that $\bar{F} = x^2 i + (3x+2)j + \sin x\, k$; Find (a) $d\bar{F}/dx$ (b) $d^2\bar{F}/dx^2$ (c) $\left|d\bar{F}/dx\right|$ (d) $\frac{d}{dx}(\bar{F}\cdot\bar{F})$ at $x = 1$

Solution

$\bar{F} = x^2 i + (3x+2)j + \sin x\, k$

a) $d\bar{F}/dx = 2xi + (3)j + \cos x\, k$

$\therefore d\bar{F}/dx = 2xi + 3j + \cos x\, k$

b) $d^2\bar{F}/dx^2 = 2i - \sin x\, k$

c) $\left|d\bar{F}/dx\right| = \sqrt{(2x)^2 + (3)^2 + (\cos x)^2}$

$= \sqrt{4x^2 + 9 + \cos^2 x}$

At $x = 1 \Rightarrow \sqrt{4(1)^2 + 9 + \cos^2(1)}$

$= \sqrt{4 + 9 + 0.999} = \sqrt{13.999} = 3.74$

d) $\frac{d}{dx}(\bar{F}\cdot\bar{F}) \rightarrow (\bar{F}\cdot\bar{F}) = [(x^2 i) + (3x+2)j + \sin x\, k] \cdot [(x^2)i + (3x+2)j + \sin x\, k]$

$= x^4 + (9x^2 + 6x + 6x + 4) + \sin^2 x$

$= x^4 + 9x^2 + 12x + 4 + \sin^2 x$

$\therefore \frac{d}{dx}(\bar{F}\cdot\bar{F}) = 4x^3 + 18x + 12 + 2\sin x \cos x$

$\frac{d}{dx}(\bar{F}\cdot\bar{F}) \Rightarrow 4(1)^3 + 18(1) + 12 + 2\sin(1)\cos(1)$ at $x = 1$

$= 4 + 18 + 12 + 2(0.0175)(0.999)$

$= 4 + 18 + 12 + 0.035$

$= 34.035$