$$\frac{dy}{dx} = -4\left[\frac{y - Be^{-6x}}{e^{-4x}}\right]e^{-4x} - 6Be^{-6x}$$

$$= -4y + 4Be^{-6x} - 6Be^{-6x} \quad \text{---(4)}$$

$$= -4y - 2Be^{-6x}, \quad \text{---(5)}$$

In eqn 5, make B the subj. of the formula

$$\frac{dy}{dx} = -4y - 2Be^{-6x}$$

$$-2Be^{-6x} = \frac{dy}{dx} + 4y$$

$$2Be^{-6x} = -\frac{dy}{dx} - 4y$$

$$B = \frac{1}{2e^{-6x}}\left[-\frac{dy}{dx} - 4y\right] \quad \text{---(6)}$$

Subst 6 into 3

$$A = y - \left[\frac{1}{2e^{-6x}}\left[-\frac{dy}{dx} - 4y\right]\right]e^{-6x}$$

$$A = y - \left[\frac{1}{2}\left[-\frac{dy}{dx} - 4y\right]\right]e^{-4x}$$

$$A = y + \frac{dy}{2dx} + 2y \quad \text{---(7)}$$

$$e^{-4x}$$

Put eqn. 6 and 7 into 2

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$$

$$= 16\left[\frac{y + \frac{dy}{2dx} + 2y}{e^{-4x}}\right]e^{-4x} + 36\left[\frac{1}{2e^{-6x}}\left[-\frac{dy}{dx} - 4y\right]\right]e^{-6x}$$

$$\frac{d^2y}{dx^2} = 16y + \frac{16dy}{2dx} + 32y + \frac{36dy}{2dx} - 72y$$

$$\frac{dy^2}{dx^2} = 16y + 8\frac{dy}{dx} + 32y - 18\frac{dy}{dx} - 72y$$

$$\frac{d^2y}{dx^2} = -24y - 10\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} + 24y + 10dy/dx = 0$$

1) Given that

$f = x^2 i + (3x+2) j + \sin x K$

find (a) $\frac{df}{dx}$ (b) $\frac{d^2f}{dx^2}$ (c) $\left|\frac{df}{dx}\right|$ (d) $\frac{d}{dx}(f \cdot f)$ at $x=1$.

2) If $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} K$, determine

(a) $\frac{dr}{dt}$ (b) $\frac{d^2r}{dt^2}$, (c) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

Answer

1) $f = x^2 i + (3x+2) j + \sin x K$

2) $\frac{df}{dx} = 2x i + 3j + \cos x K$ at $x=1$, $\frac{df}{dx} = 2i + 3j + K$.

3) $\frac{d^2f}{dx^2} = 2i + 0j - \sin x K$, at $x=1$, $\frac{d^2f}{dx^2} = 2i + 0j + 0.0175K$

4) $\left|\frac{df}{dx}\right| = \sqrt{2^2+3^2+1^2} = \sqrt{4+9+1} = \sqrt{14}$

5) $\frac{d}{dx}(f \cdot f)$

$(f \cdot f) = x^4 + 9x^2 + 12x + 4 + \sin^2 x$

$\frac{d}{dx}(f \cdot f) = 4x^3 + 18x + 12 + 2\sin x \cos x$, at $x=1$,

$\frac{d}{dx}(f \cdot f) = 4 + 18 + 12 + 0.035 = 34.035$

2) $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} K$

a) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} K$ at

b) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} K$

c) $\frac{d^2r}{dt^2}$ at $t=0 = 2i + 0j + 12K$.

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+0^2+12^2} = \sqrt{4+144} = \sqrt{148} \approx 2\sqrt{37}$