

From equation 2

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$$

$$4Ae^{4x} = -\frac{dy}{dx} - 6Be^{-6x}$$

$$Ae^{-4x} = \frac{-\frac{dy}{dx} - 6Be^{-6x}}{4}$$

$$\therefore A = \frac{-\frac{dy}{dx} - 6Be^{-6x}}{4} \quad \text{--- (4)}$$

Put equation 4 into 3

$$\frac{d^2y}{dx^2} = 16x \left[\frac{-\frac{dy}{dx} - 6Be^{-6x}}{4} \right] + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = 4 \left[\frac{-\frac{dy}{dx} - 6Be^{-6x}}{1} \right] + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} - 24Be^{-6x} + 36Be^{-6x}$$

$$\frac{d^2y}{dx^2} = -4\frac{dy}{dx} + 12Be^{-6x}$$

$$B = \left[\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right] \frac{1}{12} e^{-6x} \quad \text{--- (5)}$$

Put equation 5 into 4

$$A = \left[\frac{-\frac{dy}{dx} - 6 \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \frac{1}{12} e^{-6x}}{4} \right] \frac{1}{4} e^{-4x}$$

$$A = \left[\frac{-\frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2} - 2\frac{dy}{dx}}{4} \right] \frac{1}{4} e^{-4x}$$

$$A = \left[\frac{-\frac{3}{2} \frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2}}{4} \right] \frac{1}{4} e^{-4x} \quad \text{--- (6)}$$

Put equation 6 into 1

$$y = \left[\frac{-\frac{3}{2} \frac{dy}{dx} - \frac{1}{2} \frac{d^2y}{dx^2}}{4} \right] \frac{1}{4} e^{-4x} + \left(\frac{d^2y}{dx^2} + 4\frac{dy}{dx} \right) \frac{1}{12} e^{-6x}$$

$$y = \frac{-3}{4} \frac{dy}{dx} - \frac{1}{8} \frac{d^2y}{dx^2} + \frac{1}{24} \frac{d^2y}{dx^2} + \frac{1}{3} \frac{dy}{dx}$$

$$y = \frac{-5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2}$$

$$y = \left[-\frac{5}{12} \frac{dy}{dx} - \frac{1}{24} \frac{d^2y}{dx^2} \right] \frac{1}{12}$$