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Assignment Four.

It is discovered that $600ft^3/min$ of fresh air flows into a room containing $20000ft^3$ of air. The mixture which is made practically uniform by circulating fans, exhausted at a rate of $600ft^3/min$. If the room contains no fresh air initially.

a) develop a model for the amount of fresh air in the room at any time t

Soln,

Let $y(t)$ be the amount of air at Time t in (ft^3) in the room fresh.

$\frac{dy}{dt}$ = fresh air inflow rate - fresh air outflow rate

fresh air inflow → $600ft^3/min$

fresh air outflow → NB:- the amount flowing out of the room is a function of the amount in the room.

$\therefore \frac{600}{20000} = 0.03/min$

i.e 0.03 of $y(t)$ is the outflow = $0.03y\,ft^3/min$

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03 + 600$

$= -0.03(y - 20000)$

Thus the equation is Separable & Can be Solved.

$$\frac{dy}{(y-20000)} = -0.03dt$$

Integrate both Sides

$\ln(y - 20000) = 0.03t + C$

$y - 20000 = e^{-0.03t+C}$

$y - 20000 = e^{-0.03t} e^{C}$

recall $C = e^{C}$ = Initial Condition.

$y - 20000 = e^{-0.03t} C$ ——— (*)

At $t = 0$, $y(t) = 0$ Since the room contained no fresh air initially

$y - 20000 = Ce^{-0.03(t)}$

$0 - 20000 = C$

$C = -20000$ — (a)

put (a) in equation (*)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ — (**)

The equation above is the model for the amount of fresh air in the room

(b) Calculate the time at which 90% of the air in the room will become fresh.

$90\% = \frac{90}{100}$

$= 0.9$

$y = 0.9$ of 20000 i.e 90% of air in the room.

$= 0.9 \times 20000$

$= 18000 ft^3$

$\therefore y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = \frac{-2303}{-0.03}$

$= 76.77$

$= 77 mins$

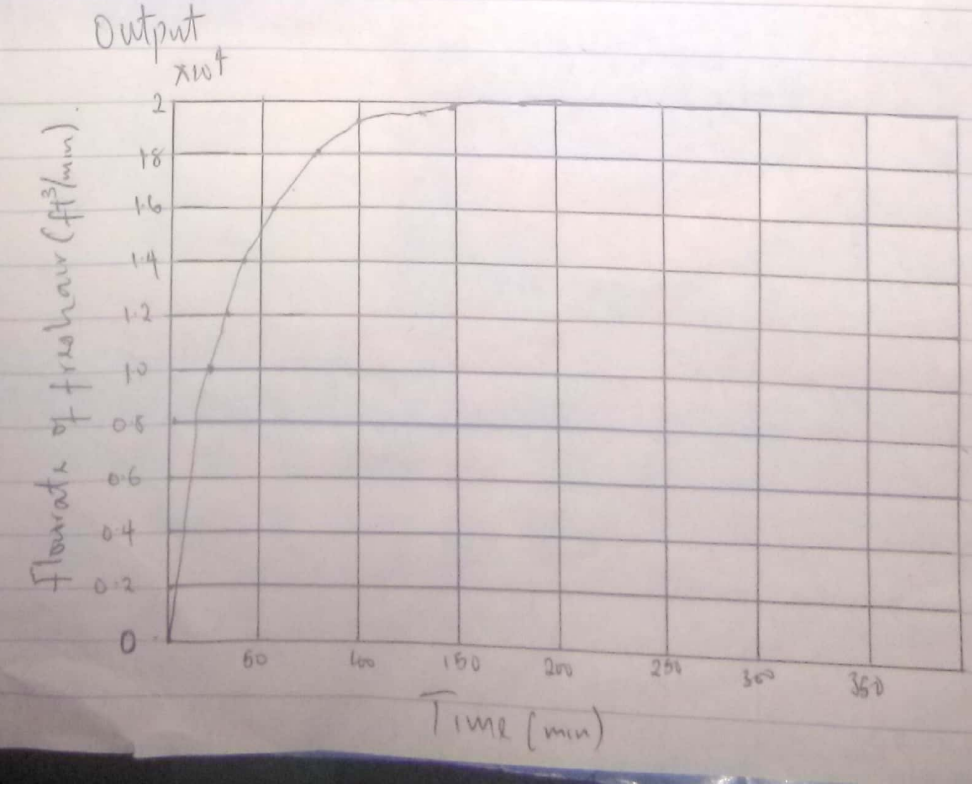
$\therefore$ The air in the room will be 90% fresh at 77minutes.

(c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6$ hr using a step of 5 min.

N/B $t = 6\text{hrs} = 6 \times 60 = 360$ mins

Answer:

```
Command window
clear all
clc
close all
Sym y, t, k
y = 20000*(1-exp(-0.05*t))
t = 0:5:360
yn = Subs(y)
plot(t, yn)
xlabel('Time(min)')
ylabel('flow-rate of fresh air(ft^3/min)')
grid on
grid minor
axis tight
```



(d) Determine the steady-state value of the amount of fresh air in the room.

Answer;

The steady-state value is $20000ft^3$ at 215mins (3hrs 35mins) of exponential approach.

(e) Comment on answer in (d)

The function shows an exponential approach to the limit of $20000ft^3$ as y increases with time. Also when the steady-state value approach $20000ft^3$ at 45mins & Continues till 360minutes (6hrs). The model discuss becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.