Awala Victor
17/ENG06/046
Mechanical Engineering
ENG 282

Assignment 1

1 a) A differential equation is a relationship between an Independent Variable (x) and dependent Variable (y) and One or more derivative of y with respect to x
  Examples: i) $\frac{dy}{dx} = R + \frac{y}{x}$
  ii) $\frac{dy}{dz} = y + \frac{y}{x}$

b) $y = Ae^{-4x} + Be^{-bx}$

c) A Second Order differential Equation

c.i) A Second Order differential equation Can be formed because it Contains 2 Constants in the degenerate Equation

ii) $y = Ae^{-4x} + Be^{-bx}$

  <u>Solution</u>

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-bx}$ ——— (1)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-bx}$ ——— (2)

  Solving eqn (1) and (2) Simultaneously
    Multiply eqn (1) by 6
$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-bx}$ ——— (3)

$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-bx}$ ——— (4)

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$

$$\therefore A = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \qquad —(5)$$

Substituting eqn.(5) into eqn (1)

$$\frac{dy}{dx} = 4\left(\frac{6\,dy/dx + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\,dy/dx + d^2y/dx^2}{2} - 6Be^{-6x}$$

Multiply through by 2.

$$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2\frac{dy}{dx} - 6\frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\,dy/dx - d^2y/dx^2}{-12e^{-6x}} = B \qquad \therefore \frac{4\,dy/dx + d^2y/dx^2}{12e^{-6x}} = B$$

Substitute A and B into the degenerate equation

$$\therefore y = \frac{6\,dy/dx + d^2y/dx^2}{-8e^{-4x}} \times e^{-4x} + \frac{4\,dy/dx + d^2y/dx^2}{+12e^{-6x}} * e^{-6x}$$

$$y = \frac{6\,dy/dx + d^2y/dx^2}{-8} + \frac{4\,dy/dx + d^2y/dx^2}{+12}$$

$$y = \frac{-72\,dy/dx - 12\,d^2y/dx^2 + 32\,dy/dx + 8\,d^2y/dx^2}{96}$$

$$y = \frac{-40\,dy/dx - 4\,d^2y/dx^2}{96}$$

$$96y = -40\frac{dy}{dx} - 4\frac{d^2y}{dx^2}$$

$$24y = -10\frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$