Assignment 6

Given: $10m_1 - 2m_2 + m_3 = 9$ — ①

$-2m_1 + 10m_2 - 2m_3 = 12$ — ②

$-2m_1 - 2m_2 + 10m_3 = 18$ — ③

Using Jacobi's iterative method:

Converting to matrix form:

$$\begin{bmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ -2 & -2 & 10 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 18 \end{bmatrix}$$

Initial value $\rightarrow \begin{bmatrix} m_1^{(0)} \\ m_2^{(0)} \\ m_3^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

First iteration:

Dividing throughout by 10 i.e eqn ①, ② and ③, we have

$m_1 - 0.2m_2 + 0.1m_3 = 0.9$

$-0.2m_1 + m_2 - 0.2m_3 = 1.2$

$-0.2m_1 - 0.2m_2 + m_3 = 1.8$

$\therefore \begin{bmatrix} m_1^{(1)} \\ m_2^{(1)} \\ m_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.2 & 0 \end{bmatrix} \begin{bmatrix} m_1^{(0)} \\ m_2^{(0)} \\ m_3^{(0)} \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 \\ 0.2 \\ 0.2 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\therefore \begin{bmatrix} m_1^{(1)} \\ m_2^{(1)} \\ m_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0.9 \\ 1.4 \\ 2.0 \end{bmatrix}$

Second iteration:

$\begin{bmatrix} m_1^{(2)} \\ m_2^{(2)} \\ m_3^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1^{(1)} \\ m_2^{(1)} \\ m_3^{(1)} \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 1.4 \\ 2.0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0.9 \\ 1.4 \\ 2.0 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\therefore \begin{bmatrix} m_1^{(2)} \\ m_2^{(2)} \\ m_3^{(2)} \end{bmatrix} = \begin{bmatrix} 1.8 \\ 2.6 \\ 3.8 \end{bmatrix}$

Third iteration:

$\begin{bmatrix} m_1^{(3)} \\ m_2^{(3)} \\ m_3^{(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} m_1^{(2)} \\ m_2^{(2)} \\ m_3^{(2)} \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 & 0.2 & -0.1 \\ 0.2 & 0 & 0.2 \\ 0.2 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 1.8 \\ 2.6 \\ 3.8 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 & 0.348 & -0.258 \\ 0.192 & 0 & 0.516 \\ 0.192 & 0.870 & 0 \end{bmatrix} \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} + \begin{bmatrix} 0.9 \\ 1.2 \\ 1.8 \end{bmatrix} \rightarrow \begin{bmatrix} 0.990 \\ 1.908 \\ 2.862 \end{bmatrix}$

$\therefore m_1 = 0.990$

$m_2 = 1.908$

$m_3 = 2.862$