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17/ENG05/023
MECHATRONICS
ENG 282
MATHEMATICS

ASSIGNMENT IV

It is discovered that 600 ft^3/min of fresh air flows into a room containing 20000 ft^3 of air. The mixture which is made practically uniform by circulating fans, is exhausted at a rate of 600 ft^3/min. If the room contains no fresh air initialy

(a) Develop a model for the amount of fresh air in the room at any time, t.

ANSWER

Let ~~Ey~~ $y(t)$ be the amount of air at time t in (ft^3) in the room.

~~fresh~~

$\left\{ \frac{dy}{dt} \right.$ = fresh air inflow rate − fresh air outflow rate. $\}$

fresh air inflow ⟶ 600 ft^3/min

fresh air Outflow ⟶ N/B : the amount

flowing out of the room is a function of the amount in the room.

$\therefore \frac{600}{20,000} = 0.03 min$

i.e 0.03 of $y(t)$ is the Outflow

$= 0.03 y ft^3$

Now; $\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600 = -0.03(y - 20000)$

Thus the equation is seperate and can be solved.

$\frac{dy}{(y - 20000)} = -0.03 dt$

Integrate both sides

$\ln(y - 20000) = -0.03t + C$

$y - 20000 = e^{-0.03t} + C$

$y - 20000 = e^{-0.03t} e^{c}$

Recall $C = e^{c}$ = initial condition

$y - 20000 = e^{-0.03t} C$ —— (1)

~~At~~ $t = 0$, $y(t) = 0$. since the room contained to fresh air, initially:

$y - 20000 = Ce^{-0.03(t)}$

$0 - 20000 = C$

(E) $C = -20000$ —— (2)

Put (a) in equation (*)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ———— (**)

The equation above is the model for the amount of fresh air in the room.

(b) Calculate the time at which 90% of the air in the room will become fresh.

$90\% = 90/100 = 0.9$

$y = 0.9$ of 20000 ie 90% of air in the room $= 0.9 \times 20000$

$= 18000 ft^3$

$\therefore y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$t = \frac{-2.303}{-0.03} = 76.77 = \underline{\underline{77 mins}}$

$\therefore$ The air in the room will be 90% fresh at 77 minutes.

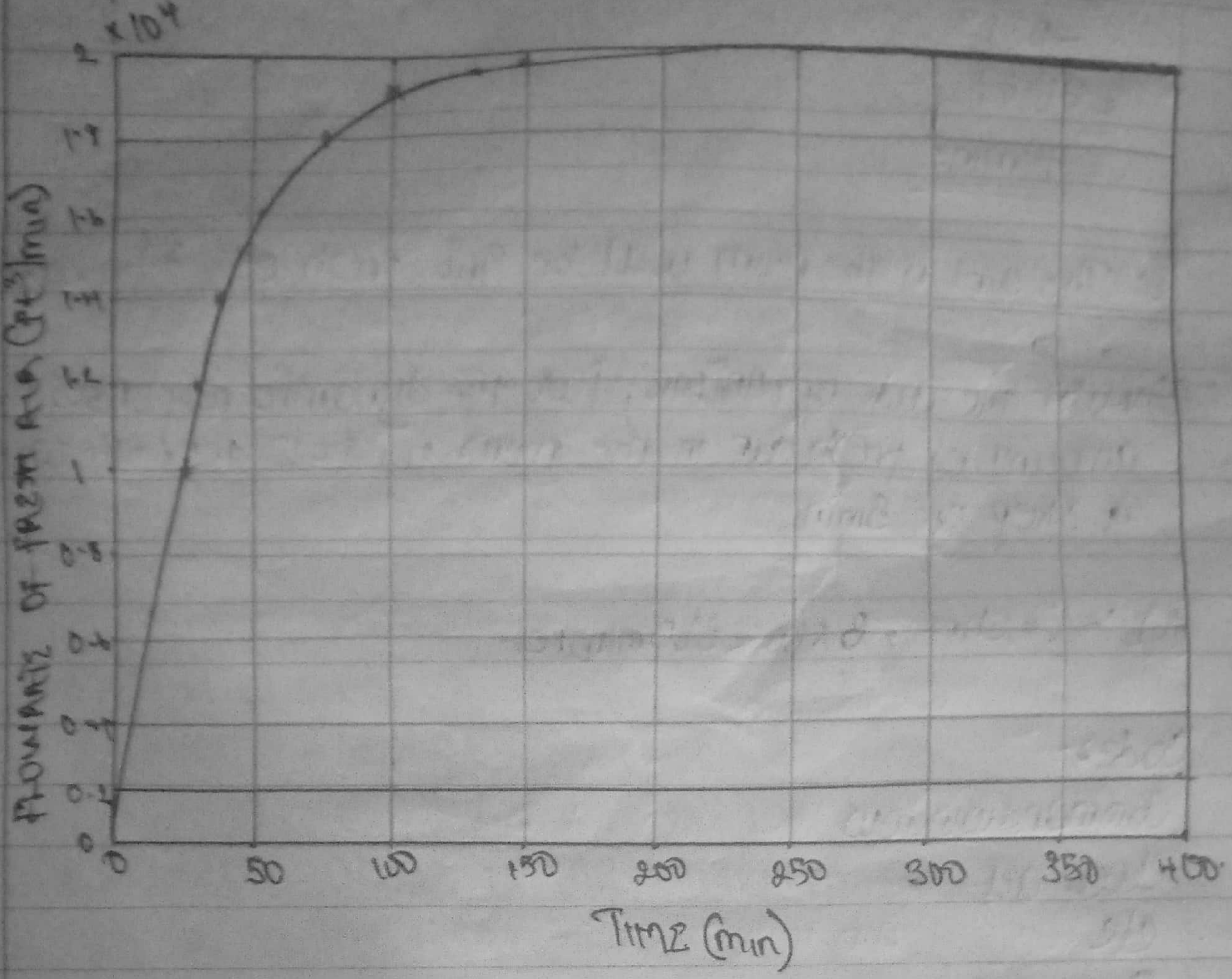
(c) With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for t = 0 to t = 6hr using a step of 5min.

N/B: – t = 6hrs = 6 × 60 = 360minutes

codes

- commandwindow
- clear all
- clc
- close all
- syms y, t, k
- y = 20000 * (1 – exp(-0.03 * t))
- t = 0 : 5 : 360
- yn = subs (y)
- plot (t, yn)
- xlabel ('TIME (MIN)')
- ylabel ('FLOWRATE OF FRESH AIR (ft^3/min)')
- grid on
- grid minor
- axis tight.

Output



(d) Determine the steady-state value of the amount of fresh air in the room

Answer.

The steady-state value is 20000ft^3/~~min~~ at 215 mins (3 hours 35 minutes) of exponential ~~...~~ approach.

(e) Comment on answer in (d)

The function shows an exponential approach to the limit of 20000ft^3 as y increases with time. Also when the steady-state value approach 20000ft^3 ~~...~~ at 215 mins and continues till 360 minutes (6 hours). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.