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### ENG282 ASSIGNMENT 1

It is discovered that  $600 \text{ ft}^3/\text{min}$  of fresh air flows into a room containing  $20000 \text{ ft}^3$  of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of  $600$  cubic feet per minute (cfm). If the room contains no fresh air initially.

- develop a model for the amount of fresh air in the room at any time  $t$ .
  - Calculate the time at which  $90\%$  of the air in the room will become fresh.
  - with the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for  $t=0$  to  $t=6$  hr using a step time of  $5$  min.
- determine the steady-state value of the amount of fresh air in the room, and comment on the result obtained in (c).

Solution:

Let  $y$  be the amount of air at time  $t$  in ( $\text{ft}^3$ ) in the room

$$\frac{dy}{dt} = \text{Air inflow rate} - \text{fresh air outflow rate}$$

Fresh Air inflow  $\rightarrow 600 \text{ ft}^3/\text{min}$

Fresh Air outflow  $\rightarrow 600$

$$2000 = 0.03 \text{ min.}$$

The amount flowing out of the room is ~~given~~ a function of the amount in the room.

$$= 0.03y \text{ ft}^3/\text{min.}$$

$$\begin{aligned} \frac{dy}{dt} &= 600 - 0.03y \\ &= -0.03y + 600 \\ &= -0.03(y - 20000) \end{aligned}$$

Equation can be resolved as

$$\frac{dy}{dt} = -0.03(y - 20000)$$

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

$$\ln(y - 20000) = -0.03t + C$$

$$\ln(y - 20000) = -0.03t + C$$

$$y - 20000 = e^{-0.03t + C}$$

$$= y - 20000 = e^{-0.03t} \cdot e^C$$

$C = e^C = \text{initial condition.}$

$$= y - 20000 = e^{-0.03t} \cdot C$$

At  $t=0$ ,  $y(t)=0$  Since no fresh air was contained in the room

initially

$$y - 20000 = C e^{-0.03(0)}$$

$$0 - 20000 = C$$

$$C = -20000$$

Re: Substituting  $C = -20000$

$$y - 20000 = e^{-0.03t} \cdot -20000$$

$$y = (e^{-0.03t} \cdot -20000) + 20000$$

$$y = 20000 - 20000e^{-0.03t}$$

$$\boxed{y = 20000(1 - e^{-0.03t})}$$

The equation above is the model for the amount of fresh air in the room

b. Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \text{ of } 20000$$

$$0.9 \times 20000 = 18000 \text{ ft}^3$$

$$\therefore 18000 = 20000(1 - e^{-0.03t})$$

$$18000 = 20000(1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$e^{-0.03t} = \ln 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{-2.3026}{-0.03}$$

$$t = 77 \text{ mins}$$

with the aid of matlab.

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```
CommandWindow
```

```
Clear all
```

```
clc
```

```
Close all
```

```
Syms t
```

$$y = 20000 * (1 - \exp(-0.03 * t))$$

```
t = 0:5:360
```

```
Yn = Subs(y)
```

```
Plot (t, Yn)
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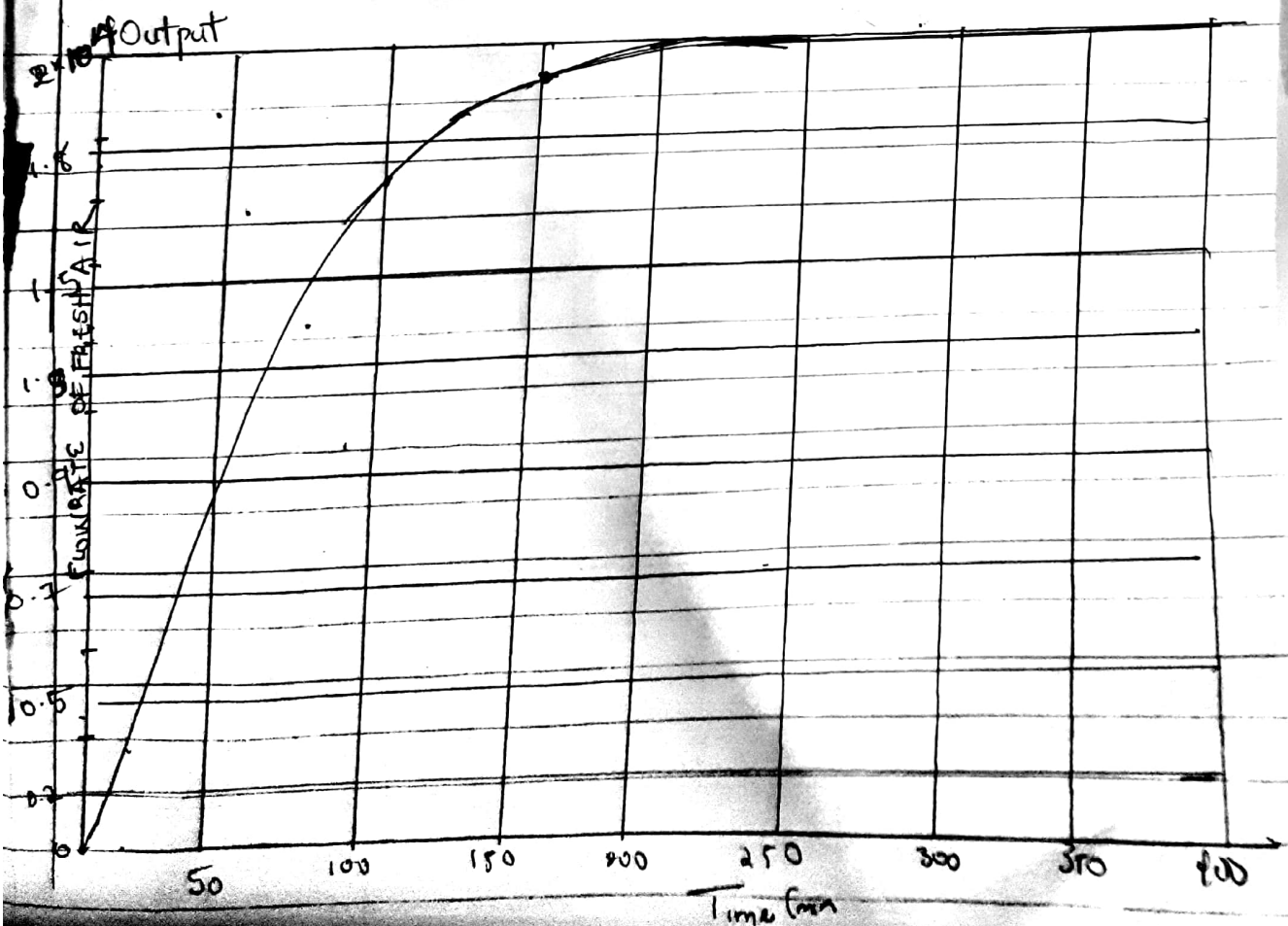
```
Xlabel ('Time (min)')
```

```
Ylabel ('Flowrate of fresh air')
```

```
grid on
```

```
grid minor
```

```
axis tight
```



d Determine the steady-state value of the amount of fresh air.  
The steady state-value is  $20000 \text{ ft}^3$  at 215 min of  
the exponential approach.

(e) Comment on the result obtained in d)

The function shows an exponential growth / approach to the  
limit of  $20000 \text{ ft}^3$  as  $y$  increase with time and the  
steady value was  $20000 \text{ ft}^3$  at 215 min ~~approximately~~ ~~whereas~~ while  
it worked for 6 hours.