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Civil Engineering

17/EN003/017

ENG 282

It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows into a room containing 20000 ft^3 of air. The mixture, which is made practically uniform by circulating fans, is exhausted at a rate of $600 \text{ ft}^3/\text{min}$ (feet per minute (cfm)). If the room contains no fresh air initially,

a) Develop a model for the amount of fresh air in the room at any time t .

To set up the model

let $y(t)$ denote the amount of fresh air at any time t . Its time rate change is

$$y' = \text{air in rate} - \text{air out rate}$$

(Balance Law)

In the out air rate are the same so volume of air in the tank is constant. The amount of fresh air flowing in is $600 \text{ ft}^3/\text{min}$. The amount flowing out is a fraction of the amount in the tank = $\frac{600}{20000} = 0.03$ of the $y(t)$ present.

In the tank flow out

the model becomes

$$dy = 600 - 0.03y$$

$$\frac{dy}{dt} = -0.03(y - 20,000)$$

Using Separating the variable

$$\frac{dy}{y - 20,000} = -0.03 dt$$

Integrating both sides

$$\frac{dy}{y-20000} = \int -0.03 dt$$

$$\ln(y-20000) = -0.03t + C$$

$$y-20000 = e^{-0.03t+C}$$

$$y-20000 = e^{-0.03t} \cdot e^C$$

$$\text{let } e^C = C$$

$$y-20000 = e^{-0.03t} \cdot C$$

$$y-20000 = Ce^{-0.03t}$$

$$y = 20000 + Ce^{-0.03t}$$

at initial point, $t=0$ amount of fresh air = 600 ppm

Substituting into eq (1)

$$600 = 20000 + Ce^{-0.03(0)}$$

$$600 - 20000 = C$$

$$C = -19400$$

\therefore eq (1) becomes

$$y = 20000 - 19400 e^{-0.03t}$$

② 90% of air will be fresh

$$= 90\% y$$

$$= \frac{90}{100} \times y = \frac{9}{10} \times 20000 = 18000$$

$$18000 = 20000 - 19400 e^{-0.03t}$$

$$18000 - 20000 = -19400 e^{-0.03t}$$

$$\frac{-2000}{-19400} = \frac{-19400}{-19400} e^{-0.03t}$$

$$0.1 = e^{-0.03t}$$

$$\ln 0.1 = \frac{-0.03t}{-0.03}$$

$$t = 75 \text{ mins}$$

~~code~~ MATLAB ~~code~~
 c. With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to 6 hrs using a step time of 5 min.

① The

Command window

clear

all

close all

Syms t

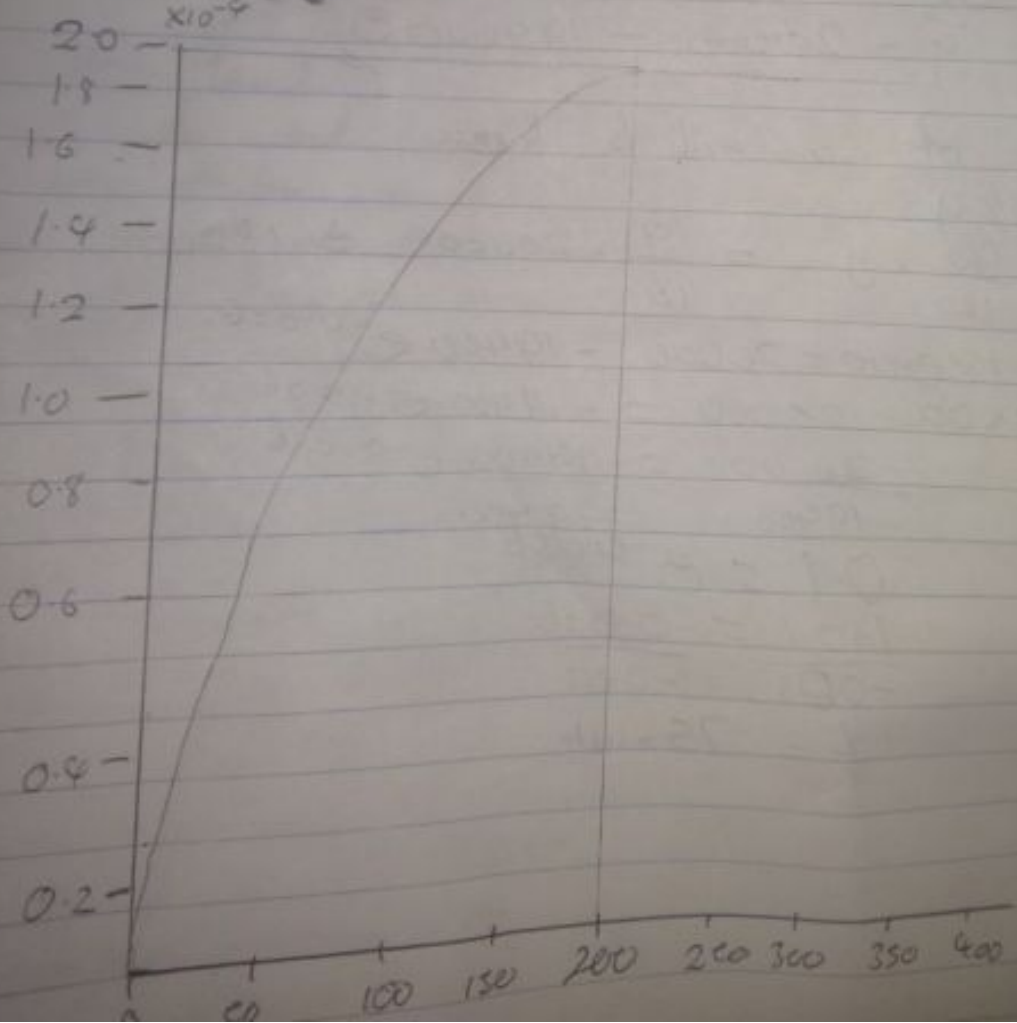
$$y = 20000 - 19400 * \exp(-0.03 * t)$$

$$t = (0:5:360)$$

$$y_n = \text{subs}(y)$$

$$y_n = \text{double}(y_n)$$

$$\text{Plot}(t, y_n)$$



amount of fresh air in the room

d

① The steady state value is 2.0×10^9

② The result shows that at 200 marks, fresh air has completely filled the room.