

Answer

$$a. f(x) = e^{-0.5x} (4-x) - 2$$

$$f'(x) \Rightarrow$$

$$\text{let } y = e^{-0.5x} \quad v = (4-x)$$

$$dy = -0.5e^{-0.5x} - 0.5e^{-0.5x} (4-x)$$

$$f'(x) = -e^{-0.5x} - 0.5e^{-0.5x} (4-x)$$

$$x_0 = 0.5 \quad \text{[Initial guess]}$$

General Newton-Raphson's formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x_0) = f(0.5) = 0.7258027407$$

$$f'(x_0) = f'(0.5) = -2.141702153$$

$$\text{root 1} \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.838896661$$

$$f(x_1) = 0.0781492979$$

$$f'(x_1) = 1.696486032$$

$$\text{root 2} \Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.8849560003$$

$$\cancel{f(x_2) = 3.23583557 \times 10^{-7}}$$

$$\cancel{f'(x_2) = -1.642200929}$$

$$f(x_2) = 1.236575203 \times 10^{-3}$$

$$f'(x_2) = -1.643060762$$

$$\text{root 3} \Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.885708605$$

$$f(x_3) = 3.23583557 \times 10^{-7}$$

$$f'(x_3) = -1.642200929$$

$$\text{root 4} \Rightarrow x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.885708802$$

$$f(x_4) = 7.845 \times 10^{12}$$

$$f'(x_4) = -1.642200704$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.885708802$$

$\therefore 0.885708802$ is the root of eqn (1.1)