

The augmented matrix is

$$A^{(1)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$\tilde{A}^{(2)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ E_2 - E_1 \Rightarrow & 0 & -1.5 & 1.5 & 0 & -2.5 & -0.5 & 6 \\ E_3 - E_1 \Rightarrow & 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ \frac{E_4}{5} - E_1 \Rightarrow & 0 & -0.6 & 1.8 & -1.2 & -2.6 & 1.2 & -4.6 \\ \frac{E_5}{-3} - E_1 \Rightarrow & 0 & -0.667 & 1.333 & -2 & -3.333 & 0 & -9.333 \\ \frac{E_6}{4} - E_1 \Rightarrow & 0 & -0.25 & 2.25 & -2.5 & -3.75 & 0.5 & -10.75 \end{array} \right]$$

Divide through  $E_3$  by 2 and swap  $E_1$  with  $E_2$

$$A^{(2)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & -1.5 & 2.5 & 0 & -2.5 & -0.5 & 6 \\ 0 & -0.6 & 1.8 & -1.2 & -2.6 & 1.2 & -4.6 \\ 0 & -0.667 & 1.333 & -2 & -3.333 & 0 & -9.333 \\ 0 & -0.25 & 2.25 & -2.5 & -3.75 & 0.5 & -10.75 \end{array} \right]$$

$$A^{(3)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ E_3 + 1.5E_2 \Rightarrow & 0 & 0 & 1.167 & -1 & -2.167 & 0.667 & -5.5 \\ \frac{E_4}{0.6} + E_2 \Rightarrow & 0 & 0 & 2.5 & -3 & -4.833 & 3 & -17.167 \\ \frac{E_5}{0.667} + E_2 \Rightarrow & 0 & 0 & 1.4999 & -3.9999 & -5.4997 & 1 & -23.493 \\ \frac{E_6}{0.25} + E_2 \Rightarrow & 0 & 0 & 8.5 & -11 & -15.5 & 3 & -52.5 \end{array} \right]$$

Divide through  $E_6$  by 2.5 and swap with  $E_3$

$$A^{(3)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 1.167 & -1 & -2.167 & 0.667 & -5.5 \\ 0 & 0 & 1.4999 & -3.999 & -5.497 & 1 & -23.493 \\ 0 & 0 & 8.5 & -11 & -15.5 & 3 & -52.5 \end{array} \right]$$

$$A^{(4)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 0.343 & 0.076 & 0.628 & 2.154 \\ 0 & 0 & 0 & -1.468 & -1.734 & -0.533 & -8.805 \\ 0 & 0 & 0 & -0.094 & 0.109 & -0.847 & 0.691 \end{array} \right]$$

$\frac{E_4}{1.167} - E_3 \Rightarrow$   
 $\frac{E_5}{1.499} - E_3 \Rightarrow$   
 $\frac{E_6}{8.5} - E_3 \Rightarrow$

Divide through by 0.343

$$A^{(4)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & -1.468 & -1.734 & -0.533 & -8.805 \\ 0 & 0 & 0 & -0.094 & 0.109 & -0.847 & 0.691 \end{array} \right]$$

$$A^{(5)} = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 0.959 & 2.194 & -0.282 \\ 0 & 0 & 0 & 0 & -1.382 & 10.842 & -13.631 \end{array} \right]$$

$\frac{E_5}{-1.468} - E_4$   
 $\frac{E_6}{-0.094} - E_4$

$$\begin{bmatrix} 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 1 & 2.288 & -0.294 \\ 0 & 0 & 0 & 0 & -1.382 & 10.157 & -13.63 \end{bmatrix}$$

$$A^{-1}(E) = \left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & 1 & -0.5 & -1 & -0.5 & 1 & -9.5 \\ 0 & 0 & 1 & -1.2 & -1.933 & 1.2 & -6.867 \\ 0 & 0 & 0 & 1 & 0.222 & -1.831 & 6.280 \\ 0 & 0 & 0 & 0 & 1 & 2.288 & -0.294 \\ 0 & 0 & 0 & 0 & 0 & -10.153 & 10.157 \end{array} \right]$$

$\frac{E_6}{-1.382} - E_5$

Backward substitution

From

$$E_6: -10.153 T_6 = 10.157$$

$$T_6 = -1.002 \approx -1$$

from

$$E_5: T_5 + 2.288 T_6 = -0.294$$

$$T_5 = -0.294 - 2.288(-1.002)$$

$$T_5 = 1.999 \approx 2$$

From

$$E_4: T_4 + 0.222 T_5 - 1.831 T_6 = 6.280$$

$$T_4 = -0.222(1.999) + 1.831(-1.002) + 6.280$$

$$T_4 = 4.002 \approx 4$$

from

$$E_3: T_3 - 1.2 T_4 - 1.933 T_5 + 1.2 T_6 = -6.867$$

$$T_3 = +1.2(4.002) + 1.933(1.999) - 1.2(-1.002) - 6.867$$

$$T_3 = 3.002 \approx 3$$

from

$$E_2: T_2 - 0.5 T_3 - T_4 - 0.5 T_5 + T_6 = -9.5$$

$$T_2 = 0.5(3.002) + 4.002 + 0.5(1.999) - (-1.002) - 9.5$$

$$T_2 = -1.996 \approx -2$$

from

$$E_1: T_1 + T_2 - 2 T_3 + T_4 + 3 T_5 - T_6 = 4$$

$$T_1 = 4 - (-1.996) + 2(3.002) - 4.002 - 3(1.999) + (-1.002)$$

$$T_1 = 0.999 \approx 1$$

$$T_1 = 1 \quad T_2 = -2 \quad T_3 = 3 \quad T_4 = 4 \quad T_5 = 2 \quad T_6 = -1$$