1/04/19 ENG 284 ASSIGNMENT IV

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Assignment IV

It is discovered that 600ft³/min of fresh air flows in a room containing 20000ft³ of air. The mixture which is made practically uniform by circulating fans, is observed at a rate of 600ft³/min. If the room contains no fresh air initially

a) develop a model for the amount of fresh air in the room at any instant

Solution

Let y(t) be the amount of air time t in (ft³) in the room

$\frac{dy}{dt}$ = air inflow rate - fresh air outflow rate

Fresh air inflow → 600 ft³/min

fresh air outflow → N/B: the amount flowing out of the room is a function of the amount in the room

$\frac{600}{20000} = 0.03$ mm

i.e 0.03 of y(t) is the outflow = 0.03y ft³

Now

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

This is the equation is separable and can be solved

$\frac{dy}{y-20000} = 0.03\,dt$

Integrate both sides

$\ln(y-20000) = 0.03t + C$

$y - 20000 = e^{-0.03t+C}$

$y - 20000 = e^{-0.03t}e^{C}$

Recall $C = e^{0}$, initial condition

$y - 20000 = e^{-0.03t}C$ ... (1)

At $t=0$, $y(t)=0$ Since the room contained no fresh air initially

$y - 20000 = Ce^{-0.03t}$

$0 - 20000 = C$

$C = -20000$ ... (2)

Put equation (2) in (1)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ... (3)

The equation above is the model for the amount of fresh air in the room

b) $\frac{90\%}{100} - \frac{9}{100} = 0.9$

$y = 0.9 \times 20000$ i.e

$= 0.9 \times 20000$

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t \approx 77 mins$

∴ The air in the room will be 90% fresh at 77 minutes.

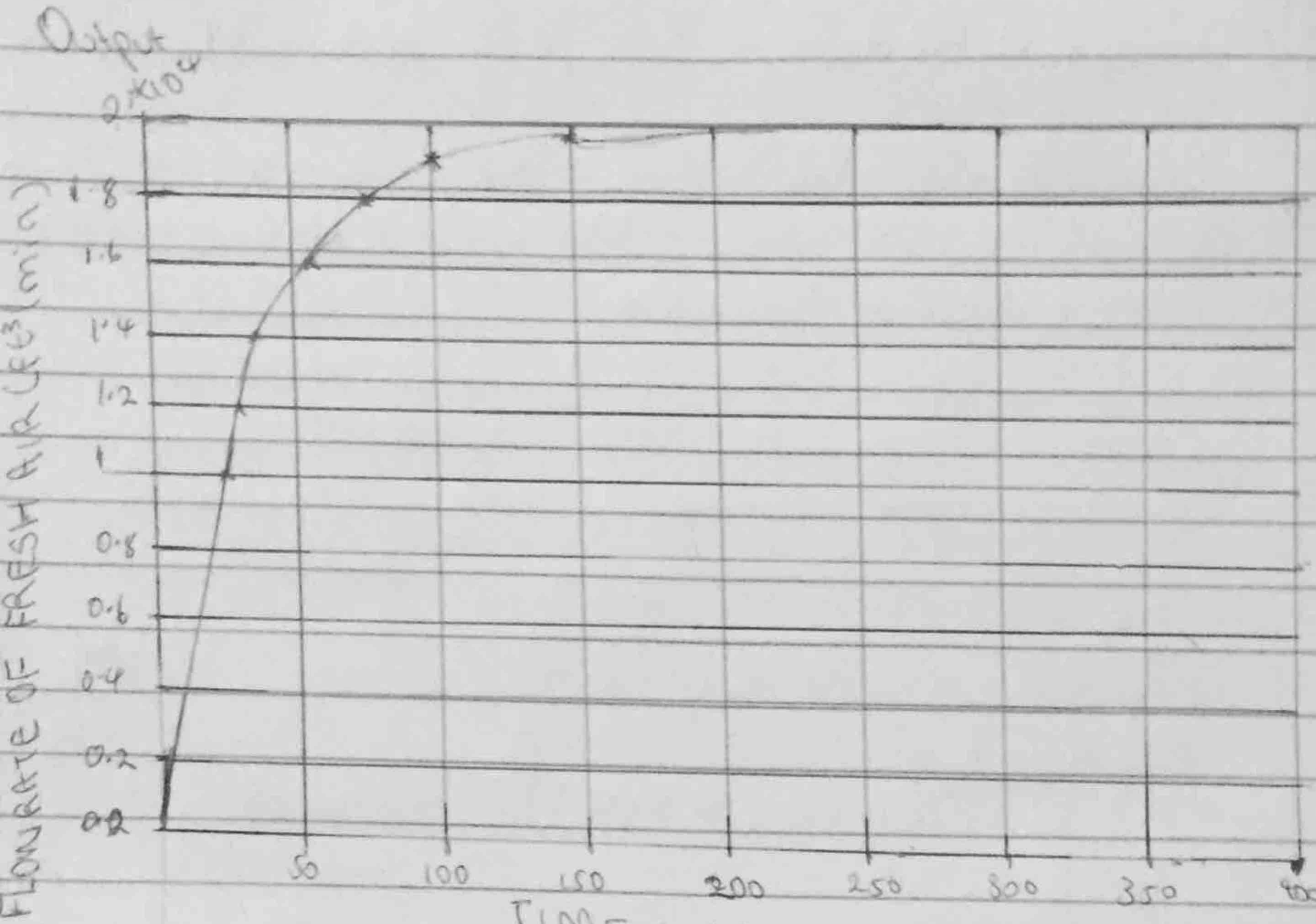
Using Matlab, the response of the amount of fresh air in the room at t:0 ~~and~~ to t:6hr with a step of 5mins

Note: t:6hrs = 360 minutes

Codes

Command window

```
Clear all
clc
Close all
Syms Y, t, k
Y = 20000*(1-exp(-0.03*t))
t = 0:5:360
Ya = subs(Y)
plot(t, Ya)
Xlabel('TIME (mins)')
Ylabel('FLOWRATE OF FRESH AIR (ft^3/min)')
grid on
grid minor
axis tight
```



Determine the Steady-state value of the amount of fresh air in a room

Steady state value is 20000 ft^3/min at 215 mins of exponential approach.

Comment on answer

The function shows an exponential approach to the limit of 200000 ft^3 as y increases with time. Also when the steady state value approach 20000 ft^3 at 215 mins and continues till 360 minutes (6 hrs). The model discusses become more realistic in pneumatic technology, although may be difficult because mixing may be imperfect.