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Fulcr 282: Engineering mathematics II.

(a) A differential equation is a relationship between dependent and independent variable and one or more derivatives of the dependent variable with respect to independent variable.

i)  $x \frac{dy}{dx} = 2x - y$

ii)  $y = x \frac{dy}{dx} - x^2/2 \frac{d^2y}{dx^2}$

(b) Second order equation

i) it is a second order equation because it has two arbitrary constants.

iii)  $y = Ae^{-4x} + Be^{-6x}$  ----- (1)

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x}$  ----- (2)

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x}$  ----- (3)

$A = \frac{y - Be^{-6x}}{e^{-4x}}$  ----- (4)

Substituting eqn (4) into (2)

$\frac{dy}{dx} = -4 \left( \frac{y - Be^{-6x}}{e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$

$= -4y + 4Be^{-6x} - 6Be^{-6x}$

$$= -4y - 2Be^{-6x} \quad \text{--- --- --- (5)}$$

Making B the subject of the formula

$$\frac{dy}{dx} = -4y - 2Be^{-6x}$$

$$\frac{dy}{dx} + 4y = -2Be^{-6x}$$

$$\frac{-1}{2e^{-6x}} \left[ \frac{dy}{dx} + 4y \right] = B \quad \text{--- --- --- (6)}$$

Substituting equation 6 into 4

$$A = \frac{y - Be^{-6x}}{e^{-6x}}$$

$$A = y - \left[ \frac{-1}{2e^{-6x}} \left[ \frac{dy}{dx} + 4y \right] \right] e^{-6x}$$

$$A = y + \frac{\frac{dy}{dx} + 4y}{2e^{-6x}} \quad \text{--- --- --- (7)}$$

Substituting eqn 7 and 6 into 8

$$\frac{d^2y}{dx^2} = 16Ae^{-6x} + 36Be^{-6x}$$

$$= 16 \left[ y + \frac{\frac{dy}{dx} + 4y}{2e^{-6x}} \right] e^{-6x} + 36 \left[ \frac{-1}{2e^{-6x}} \left[ \frac{dy}{dx} + 4y \right] \right] e^{-6x}$$

$$\frac{d^2y}{dx^2} = 16y + \frac{16dy}{2dx} + 32y - \frac{36dy}{2dx} - 72y$$

$$\frac{d^2y}{dx^2} = 16y + \frac{18dy}{dx} + 32y - \frac{18dy}{dx} - 72y$$

$$= -24y - \frac{18dy}{dx} = \frac{d^2y}{dx^2} + 24y + \frac{18dy}{dx} = 0$$