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Solution

) Develop a model for the amount of fresh air in the room at any time 't'

Solution

a) Let 'y(t)' be the amount of air at time t. In (ft³) in the room

$\frac{dy}{dt}$ = fresh air inflow rate − fresh air outflow rate

fresh air inflow ⇒ 600 ft³/min
fresh air outflow → N/B :- the amount flowing out of the room is a fraction of the amount in room

$\therefore \frac{600}{20000} = 0.03$ min

i.e 0.03 of y(t) is out flow = 0.03y ft³

Now, $\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

Thus the equation is separable and can be solved

$\frac{dy}{(y - 20000)} = -0.03dt$

Integrating both sides w.r.t t

$\ln(y - 20000) = -0.036t + C$

$y - 20000 = e^{-0.036t + C}$

$y - 20000 = e^{-0.03t} \cdot e^{C}$

Let $e^{C} = C$

$y = y - 20000 = e^{-0.03t} \cdot C$

$y - 20000 = C \cdot e^{-0.03t}$

At $t = 0$, $y(t) = 0$ since the room had no fresh air

$\therefore y - 20 \quad y_0 = C \cdot e^{-0.03(0)} + 20000 = 0$

$y_0 = C + 20000$

$C = -20000$

$\therefore y = 20000 - 20000\, e^{-0.03t}$

$y = 20\,000\,(1 - e^{0.036})$ ... Answer to the model

b) Calculate the time at which 90% of the air in the room will be fresh

90% = 90/100

y = 0.9 of 20000

= 18000

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$0.03t = \ln 0.1$

$t = \frac{\ln 0.1}{0.03}$

$t = 76.72 \text{ min}$

c) With matlab codes

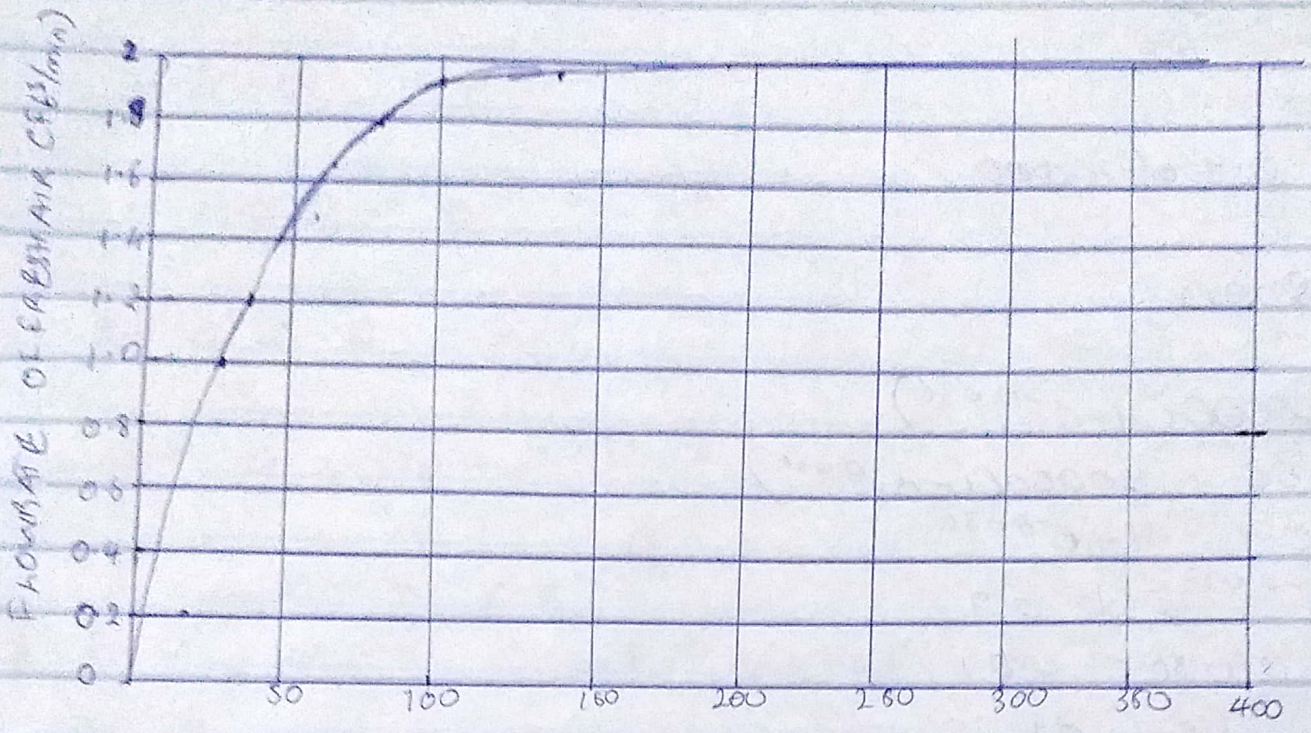
using 6 hrs = 360 min

Solution

```
command window
clear
clc
close all
syms y, t, k
y = 20000 * (1 - exp(-0.03 * t))
t = 0:5:360
yn = subs(y)
plot(t, yn)
xlabel('Time (min)')
ylabel('Flowrate of FRESH AIR (ft^3/min)')
grid on
```

Said answer

OUTPUT



Determine the steady state value of the amount of fresh air in the room

The steady - state value is 20000 ft³/min at 215 mins of exp. approach

Comment on answer

The function shows an exponential approach to the limit of 20000 ft³/min as increases with time. Also when the steady state value approach 20000 ft³/min and continues till 360 min (6 hrs). The model devised becomes more true in practice, although it may be difficult