Name: Mba Jonah Abali
Department: Computer Engineering
Matric Number: 17/engHS01/187

Assignment IV

It is discovered that 600ft³/min of fresh air flows into a room containing 20000ft³ of air. The mixture which is made practically uniform by circulating fans, is exhausted at the rate of 600ft³/min. If the room contains no fresh air initially (a) develop a model for the amount of fresh air in the room at any time, t.

Answer:

Let y(t) be the amount of air at any time t in ft³ in the room.

$\frac{dy}{dt}$ → fresh air inflow rate − fresh air outflow rate

Fresh air inflow → 600ft³/min

Fresh air outflow → Note: The amount flowing out of the room is a function of the amount in the room

$\therefore \frac{600}{20000} = 0.03$ min

i.e 0.03 of y(t) is the outflow $= 0.03y$ ft³/min

Now;

$\frac{dy}{dt} = 600 - 0.03y$

$= -0.03y + 600$

$= -0.03(y - 20000)$

This equation can be seperated and integrated;

$\frac{dy}{(y-20000)} = -0.03dt$

Find the integral of both sides

$\ln(y-20000) = -0.03t + C$

$y - 20000 = e^{(-0.03t+C)}$

$y - 20000 = e^{-0.03t} \cdot e^{C}$

Recall $C = e^{C}$ = initial equation

$\therefore y - 20000 = e^{-0.03t} \cdot C$ ---------- (1)

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially,

Put $y=0; t=0$ in eqn(I)

$y - 20000 = e^{-(0.03t)} \cdot c$

$0 - 20000 = e^{0} \cdot c$

$0 - 20000 = 1(c)$

$c = -20000$ ---------- (2)

Put eqn (2) in eqn(I)

$y = 20000 - 20000e^{-0.03t}$

$y = 20000(1 - e^{-0.03t})$ ---------- (3)

Equation(3) above is the model for the amount of fresh air in the room.

6 Calculate the time at which 90% of the air in the room will become fresh.

$90\% = \frac{90}{100} = 0.9$

$y = 0.9 \times 20{,}000$, i.e. 90% of air in the room

$= 18000 ft^3$

$y = 20000(1 - e^{-0.03t})$

$18000 = 20000(1 - e^{-0.03t})$

$0.9 = 1 - e^{-0.03t}$

$e^{-0.03t} = 1 - 0.9$

$e^{-0.03t} = 0.1$

$-0.03t = \ln(0.1)$

$t = \frac{\ln(0.1)}{-0.03}$

$= \frac{-2.303}{-0.03}$

$= 76.77 mins$

$\approx 77 mins$

c With the aid of Matlab, plot the dynamic response of the amount of fresh air in the room for $t = 0$ to $t = 6 hrs$, using a step of 5min

Note: $t = 6 hrs$

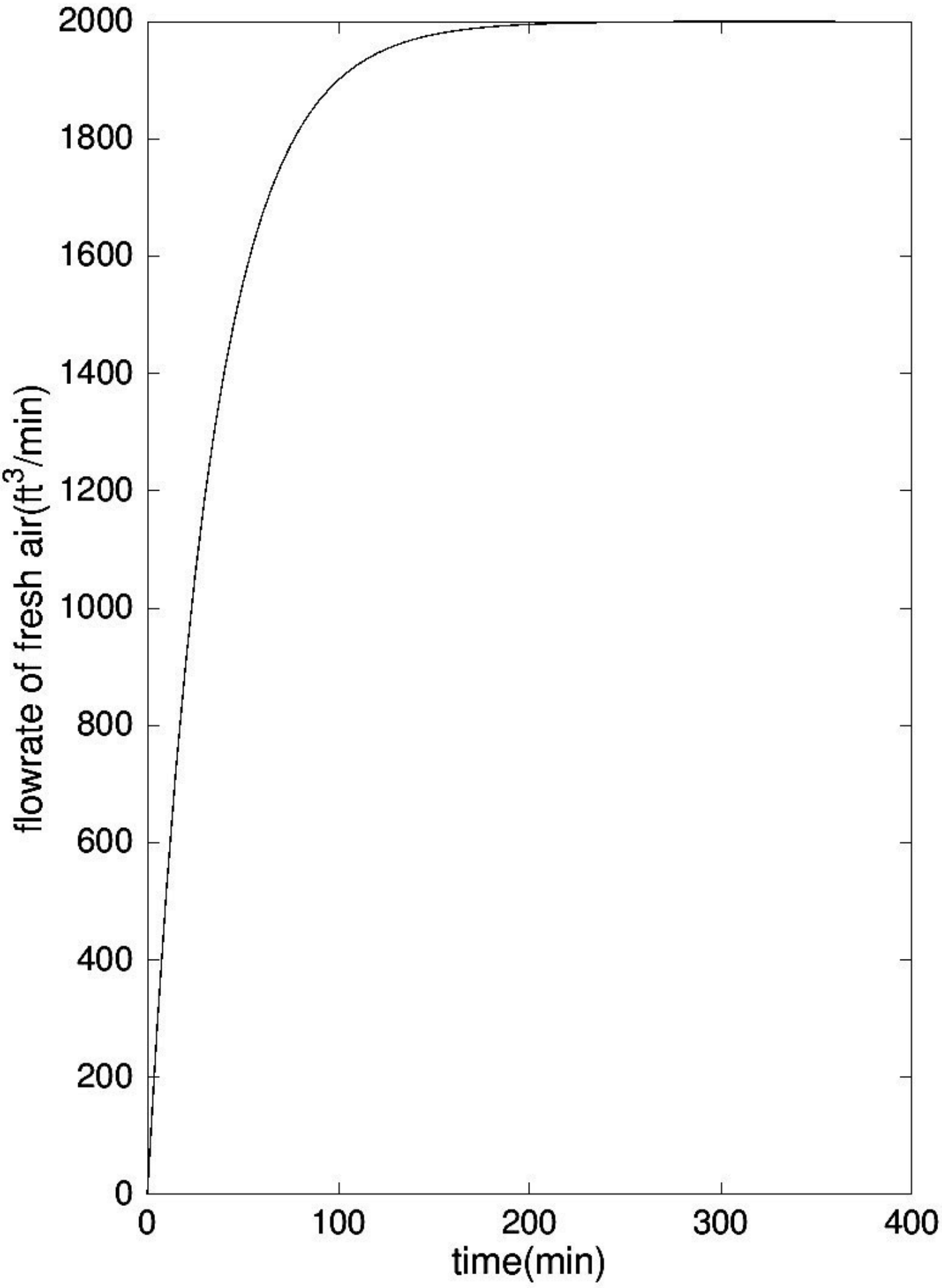
$= 6 \times 60s$

$= 360 mins$

Solution

```
commandwindow
clear all
clc
close all
syms y,t
y = 20000 * (1- exp(-0.03*t))
t = 0:5:360
yn = subs(y)
Plot (t, yn)
Xlabel ('Time(min)')
Ylabel ('flowrate of fresh air (ft^3/min)')
Grid On
Grid minor
Axis tight
```

Output



(d) Determine the steady-state value of the amount of fresh air in the room.

A: The steady-state value is $2000ft^3$ at 215mins (3hr and 35mins) at exponential approach.

(e) Comment on answer in (d)

The functions above shows an exponential approach to the limit of $2000ft^3$ as y increases with time. Also, when the steady state value approaches $2000ft^3$ at 215minutes and continues till 300mins (6hrs). The model discussed becomes more realistic in pneumatic technology, although maybe difficult because mixing may be imperfect.