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 IT/ENG 1/003
 CHEMICAL ENGINEERING
~~PHYSICS~~ MATHS

It is discovered that $600 \text{ ft}^3/\text{min}$ of fresh air flows into room
 20000 ft^3 of air. The mixture which is mixed practically
 uniform by is exhausted at a rate of 0.03 of $600 \text{ ft}^3/\text{min}$
 if the room contains no fresh air initially

- (a) develop a model for the amount of fresh air in the room
 at any time t
 (b) calculate time at which 90% of the air in the room will
 become fresh
 (c) with the aid of matlab plot the dynamic response of the amount
 of fresh air intake for $t=0$ to $t=6 \text{ hrs}$ with step time of 5 min

SOLUTION

let $y(t)$ be the amount of air at any time t in ft^3 within
 room

$$\frac{dy}{dt} = \text{fresh air inflow rate} - \text{fresh air outflow rate}$$

fresh air inflow = $600 \text{ ft}^3/\text{min}$

fresh air outflow NB: The amount flowing out of the room
 is a function of the amount in room

Hence $\frac{600}{20000} = 0.03 \text{ min}$

$\therefore 0.03$ of $y(t)$ is the outflow = $0.03y \text{ ft}^3/\text{min}$
 now,

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$\frac{dy}{dt} = 0.03(y - 20000)$$

This can be simplified as

$$\frac{dy}{(y - 20000)} = -0.03 dt$$

$$(y - 20000)$$

$$\ln(y-20000) = -0.03t + c$$

$$y-20000 = e^{-0.03t+c}$$

$$y-20000 = e^{-0.03t} \cdot e^c$$

Let $e^c = y_0$

$$y-20000 = e^{-0.03t} \cdot y_0$$

$$y-20000 = y_0 e^{-0.03t} \quad \text{--- (1)}$$

At $t=0$, $y(t) = 0$ as the room contained no fresh air initially

Hence eqn (1) becomes

$$y-20000 = e^{-0.03t} \cdot y_0$$

$$0-20000 = e^0 \cdot y_0$$

$$y_0 = 20000$$

Hence the $y_0 = 20,000$ into eqn (1)

$$y = 20,000 (1 - e^{-0.03t})$$

$$y = 20000 (1 - e^{-0.03t}) \quad \text{--- Model for the}$$

amount of fresh air

(b) ~~Calculate~~

$$90\% = \frac{9}{10} = 0.9$$

$$y = 0.9 \times 20,000$$

$$= 18,000 \text{ pt}^3$$

$$y = 20000 (1 - e^{-0.03t})$$

$$\therefore 18000 = 20000 (1 - e^{-0.03t})$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$\frac{-0.03t}{-0.03} = \frac{\ln(0.1)}{-0.03}$$

$$e^{-7} = 0.77 \text{ min}$$

$$\begin{aligned} \textcircled{c} \quad t &= 6 \text{ hrs} \\ &= 6 \times 60 \text{ s} \\ &= 360 \text{ mins} \end{aligned}$$

MATLAB n-file

command window

clear

clc

close all

$$t = 0 : 5 : 360$$

$$y = 2000 * (1 - \exp(-0.03t))$$

$y_n = \text{subs}(y)$

plot(t, y_n)

grid on

grid minor

xlabel('Time (secs)')

ylabel('Y (litres/min)')



$$\textcircled{d} \text{ Steady value} = 20000$$